

Pure 27 Partial Fractions.

Section 1:

1) a) $u_5 = 23$, $S_{10} = 240$
 $a + 4d = 23$ $240 = 10a + 45d$
 $\Rightarrow d = 2$, $a = 15$

b) $S_{60} = 30(30 + 59 \times 2) = 4440$

2) $-2 + 2 + 6 + \dots = 720$ $a = -2$ $d = 4$ $S_n = 720$
 $720 = \frac{n}{2}(-4 + (n-1)4)$
 $1440 = n(4n - 8)$
 $4n^2 - 8n - 1440 = 0$
 $n = 20$, ~~$n = 18$~~

3) a) $u_2 = ar = 0.5$ $u_5 = ar^4 = 32 \Rightarrow r^3 = \frac{32}{0.5} = 64$, $r = 4$ $a = \frac{1}{8}$

b) $u_n < 10000$ $\frac{1}{8}(4)^{n-1} < 10000$
 $4^{n-1} < 80000$
 $(n-1) \log 4 < \log 80000$
 $n < \frac{\log 80000}{\log 4} + 1 \Rightarrow n < 9.14$
so 9 terms.

4) $r = 0.55$ $S_\infty = 40$

a) $40 = \frac{a}{0.45}$, $a = 18$

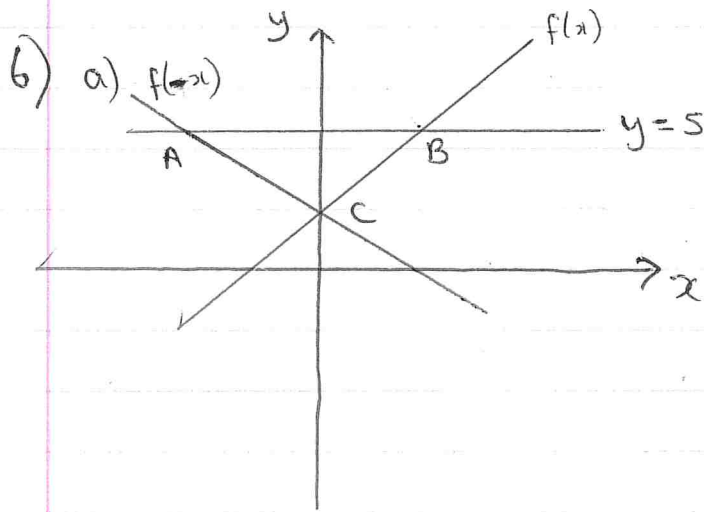
b) $18(0.55)^{n-1} < 0.001$
 $(0.55)^{n-1} < \frac{0.001}{18}$

$(n-1) \log 0.55 < \log \left(\frac{0.001}{18} \right)$

$n-1 > \frac{\log \left(\frac{0.001}{18} \right)}{\log 0.55}$

$n > 17.39$ so 18th term is the first to fall below 0.001.

- 5) a) reflection in y -axis
 b) reflection in x -axis
 c) $f(-x) = -3x + 2$, $-f(x) = -3x - 2$



- b) $3x + 2 = 5$, $x = 1$ $C(1, 5)$
 $-3x + 2 = 5$, $x = -1$ $A(-1, 5)$
 $3x + 2 = -3x + 2 \Rightarrow x = 0$ $B(0, 2)$
 c) Symmetrical in y -axis.

7) a) $(x+3)(x+2)$ b) x^2 c) $x(x+1)(x+2)$

8) a) $2(-3)^3 + (-3)^2 - 13(-3) + 6 = 0 \therefore (x+3)$ is a factor of $f(x)$

b)

$$\begin{array}{r} 2x^2 - 5x + 2 \\ x+3 \overline{) 2x^3 + x^2 - 13x + 6} \\ \underline{2x^3 + 6x^2} \\ -5x^2 - 13x + 6 \\ \underline{-5x^2 - 15x} \\ 2x + 6 \\ \underline{2x + 6} \\ r = 0 \end{array}$$

$(x+3)(2x^2 - 5x + 2)$
 $(x+3)(2x-1)(x-2)$

c) $\frac{2x^3 + x^2 - 13x + 6}{x^2 - x} = \frac{(x+3)(2x-1)(x-2)}{x(x-2)} = \frac{(x+3)(2x-1)}{x}$

$= \frac{2x^2 + 5x - 3}{x} = 2x + 5 - 3x^{-1}$

Section 2:

1 a) $8 = A(x+3) + B(x-1)$ $x=1 \Rightarrow A=2$ ✓

$x=-3 \Rightarrow B=-2$ ✓

$$\frac{8}{(x-1)(x+3)} = \frac{2}{x-1} - \frac{2}{x+3}$$
 ✓

b) $5x+7 = A(x+1) + Bx$ $x=0 \Rightarrow A=7$ ✓

$x=-1 \Rightarrow B=-2$ ✓

$$\frac{5x+7}{x(x+1)} = \frac{7}{x} - \frac{2}{x+1}$$
 ✓

c) $1-3x = A(2x+1) + B(3x+4)$ $x=-\frac{1}{3} \Rightarrow A=-3$ ✓

$x=-\frac{1}{2} \Rightarrow B=1$ ✓

$$\frac{1-3x}{(2x+1)(3x+4)} = \frac{-3}{3x+4} + \frac{1}{2x+1}$$
 ✓

d) $2x+10 = A(2x+3) + B(4x-1)$

$x=\frac{1}{4} \Rightarrow A=3$ ✓

$x=-\frac{1}{2} \Rightarrow B=-1$ ✓

$$\frac{2(x+5)}{8x^2+10x-3} = \frac{3}{4x-1} - \frac{1}{2x+3}$$
 ✓

2 a) $8x+14 = A(x+1)(x+3) + B(x-2)(x+3) + C(x-2)(x+1)$

$x=2 \Rightarrow A=2$ ✓

$x=-1 \Rightarrow B=-1$ ✓

$x=3 \Rightarrow C=-1$ ✓

b) $2x^2-6x+20 = A(x-2)(x-6) + B(x+1)(x-6) + C(x+1)(x-2)$

$x=-1 \Rightarrow A=\frac{4}{3}$ ✓

$x=2 \Rightarrow B=-\frac{4}{3}$ ✓

$x=6 \Rightarrow C=2$ ✓

$$3 \text{ a) } \frac{9}{(x-2)(x+1)^2} = \frac{A}{x-2} + \frac{B}{x+1} + \frac{C}{(x+1)^2} \quad \checkmark$$

$$9 = A(x+1)^2 + B(x+1)(x-2) + C(x-2)$$

$$x=2 \Rightarrow A=1 \quad \checkmark$$

$$x=-1 \Rightarrow C=-3 \quad \checkmark$$

$$x=0 \Rightarrow B=-1 \quad \checkmark$$

$$b) \frac{5x^2+3x-20}{x^2(x+4)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+4} \quad \checkmark$$

$$5x^2+3x-20 = Ax(x+4) + B(x+4) + C(x^2)$$

$$x=0 \Rightarrow B=-5 \quad \checkmark$$

$$x=-4 \Rightarrow C=3 \quad \checkmark$$

$$\text{coefficient of } x^2: 5 = A + C \Rightarrow 5 = A + 3 \Rightarrow A = 2 \quad \checkmark$$

$$4 \text{ a) } x^2 = A(x-2)(x-6) + B(x-6) + C(x-2) \quad \checkmark$$

$$x=2 \Rightarrow B=-1 \quad \checkmark$$

$$x=6 \Rightarrow C=9 \quad \checkmark$$

$$\text{coeff of } x^2: 1 = A \quad \checkmark$$

$$b) x^2+2x+9 = A(x-1)(x+5) + B(x+5) + C(x-1) \quad \checkmark$$

$$x=1 \Rightarrow B=2 \quad \checkmark$$

$$x=-5 \Rightarrow C=-4 \quad \checkmark$$

$$A=1 \quad \checkmark$$

$$5a) \quad 5 - 12x = A(4 + 3x) + B(1 + 6x)$$

$$x = -4/3, \quad 21 = -7B, \quad B = -3 \quad \checkmark$$

$$x = -1/6, \quad 7 = \frac{7}{2}A, \quad A = 2 \quad \checkmark$$

$$b) \quad (1 + 6x)^{-1} = 1 - 6x + \frac{(-1)(-2)}{2} 36x^2 + \dots = 1 - 6x + 36x^2 + \dots \quad \checkmark \quad \checkmark$$

$$\begin{aligned} (4 + 3x)^{-1} &= \frac{1}{4} \left(1 + \frac{3}{4}x\right)^{-1} = \frac{1}{4} \left(1 - \frac{3}{4}x + \frac{(-1)(-2)}{2} \frac{9}{16}x^2 + \dots\right) \\ &= \frac{1}{4} - \frac{3}{16}x + \frac{9}{64}x^2 + \dots \end{aligned}$$

$$c) \quad 2(1 - 6x + 36x^2) - 3\left(\frac{1}{4} - \frac{3}{16}x + \frac{9}{64}x^2\right) \quad \checkmark$$

$$= \frac{5}{4} - \frac{183}{16}x + \frac{4581}{64}x^2 \quad \checkmark$$

$$d) \quad |6x| < 1 \quad \therefore |x| < \frac{1}{6} \quad \left| \frac{3}{4}x \right| < 1 \quad \therefore |x| < \frac{4}{3} \quad \checkmark \quad \text{find both}$$

so the most restrictive is $|x| < \frac{1}{6}$ \checkmark correct conclusion

