

- Modulus Function and Graph Transformations SOLUTIONS

Section 1

1.

a) 6, 8, 10

b) $u_{42} = 6 + 41 \times 2 = 88$

c) $u_n = 6 + (n-1)2 = 2n + 4$

d) $S_{14} = \frac{14}{2} \{ 12 + 13 \times 2 \} = 7 \times 38 = 266$

2.

a) $\sum_{r=1}^4 (2r+3) = 5 + 7 + 9 + 11$

b) $\sum_{r=0}^3 2 \times 3^r = 2 + 6 + 18 + 54$

c) $\sum_{r=1}^4 (-1)^r r^2 = -1 + 4 + -9 + 16$

3.

a) 2, 7, 27, 107 divergent

b) 3.2, 3.04, 3.008, 3.0016 convergent

4.

a) 2, 6, 18 $r=3$

b) 54, 162

c) $u_{13} = 2 \times 3^{12} = 1,062,882$

5.

$$a) \begin{cases} u_2 = ar = 9 \\ u_5 = ar^4 = 1.125 \\ u_2 = ar = 9 \end{cases} \Rightarrow r^3 = \frac{1.125}{9} = 0.125$$

$$r = 0.5$$

$$b) a(0.5) = 9 \Rightarrow a = 18$$

$$c) S_{\infty} = \frac{18}{1-0.5} = 36$$

6.

$$i) x^2 + y^2 - 10x - 8y + 21 = 0$$

$$(x-5)^2 - 25 + (y-4)^2 - 16 + 21 = 0$$

$$(x-5)^2 + (y-4)^2 = 20 \Rightarrow \text{centre } (5, 4)$$

$$b) y=0 \Rightarrow (x-5)^2 + 16 = 20$$

$$(x-5)^2 = 4 \quad x = 5 \pm 2 \Rightarrow x = 7 \text{ or } 3 \Rightarrow (7, 0)$$

7.

$$3x^2 - x - 4 = 0 \text{ at } A \text{ \& } C, \text{ and } x=0 \text{ at } B$$

$$(3x-4)(x+1) = 0$$

$$x = 4/3 \text{ or } -1$$

$$A(-1, 0) \quad B(0, -4) \quad C(4/3, 0)$$

Section 2

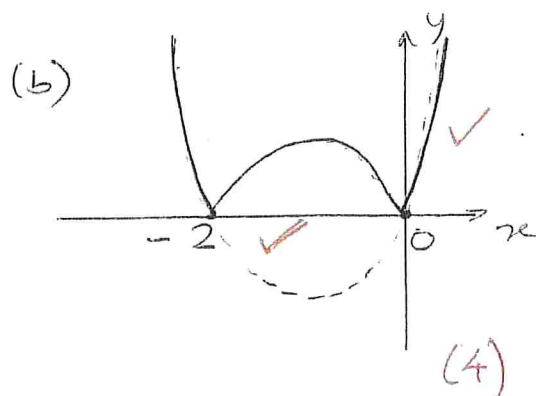
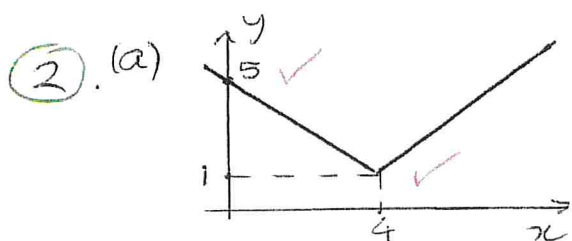
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SECTION 2 SOLUTIONS

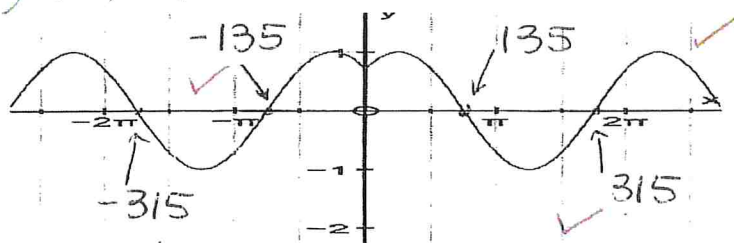
① (a) → horizontal stretch by s.f. = -1
 (a reflection in the y-axis) ✓
 ↑ vertical stretch, s.f. = 3. ✓

(b) → horizontal translation by 3 to left ✓
 ↑ vertical stretch, s.f. = 2. ✓

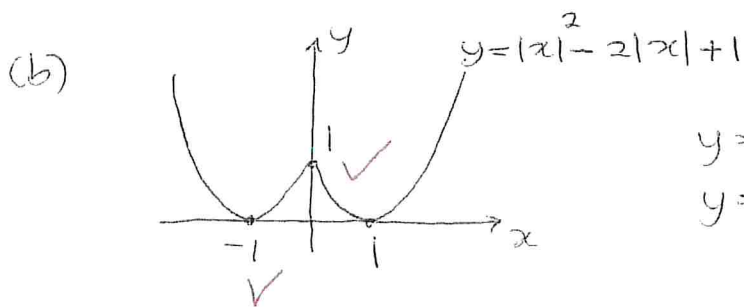
(c) → horizontal stretch, s.f. = $\frac{1}{2}$ ✓
 ↑ vertical stretch, s.f. = -1 ✓
 (a reflection in the x-axis). (6)



③ (a) $y = \sin(|x| + 45)$



sketch $y = \sin(x + 45)$
 remove curve for
 negative x values
 + reflect remaining
 curve in y-axis



$$y = |x|^2 - 2|x| + 1$$

$$y = x^2 - 2x + 1 \quad (5)$$

$$x^2 - 2x + 1 = 0$$

$$(x-1)^2 = 0 : x = 1$$

(1, 0)

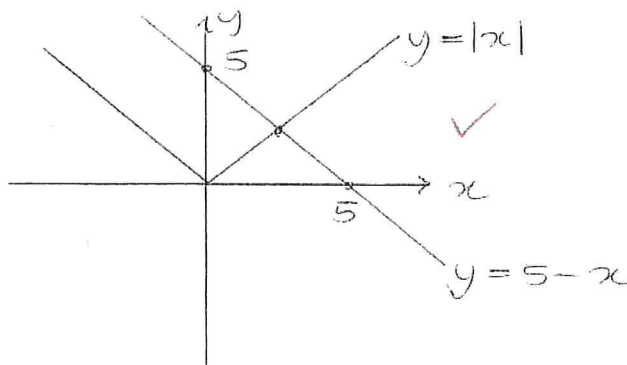
when $x = 0$, $y = 1$. (0, 1)

④ (a) $x = \frac{1}{2}x + 4$
 $\frac{1}{2}x = 4$
 $x = 8 \checkmark$

OR $-x = \frac{1}{2}x + 4$
 $-x - 0.5x = 4$
 $-\frac{3}{2}x = 4$
 $x = -\frac{8}{3} \checkmark$

(b) $x = 5 - x$
 $2x = 5$
 $x = 2.5 \checkmark$

OR $-x = 5 - x$
 $-x + x = 5$
 $0 = 5$
 \times no solutions.

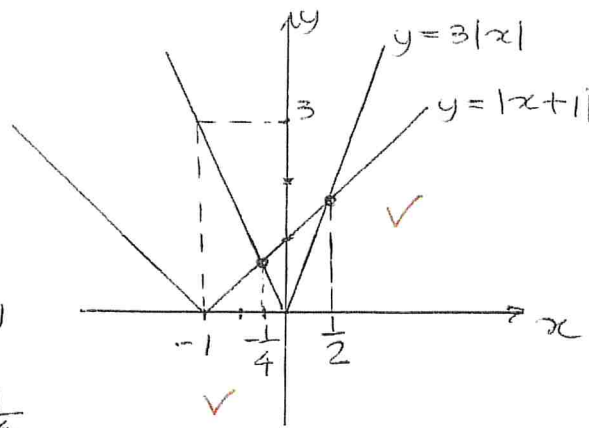


Only one answer as $y = |x|$ and $y = 5 - x$ cross only once. \checkmark

(5)

⑤ $3|x| > |x+1|$
 $3|x| = |x+1|$

$3x = x+1$ OR
 $2x = 1$ $3x = -x-1$
 $x = \frac{1}{2}$ $4x = -1$
 $x = -\frac{1}{4}$

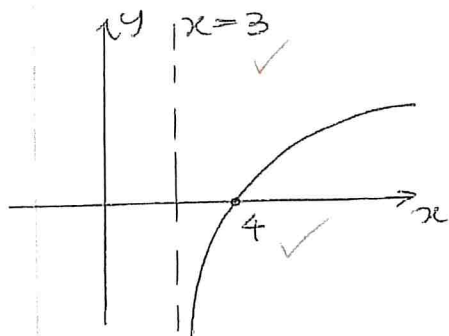


There are 3 regions to consider:

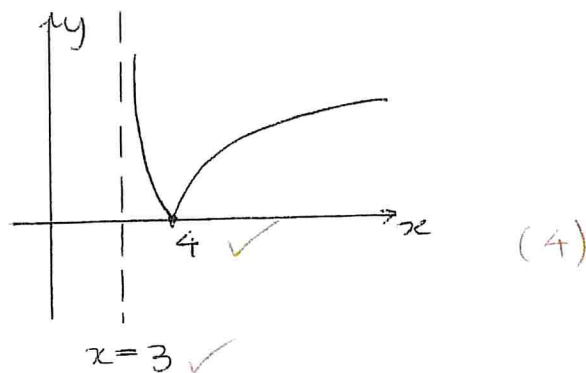
$x < -\frac{1}{4}$	$-\frac{1}{4} < x < \frac{1}{2}$	$x > \frac{1}{2}$
$3 x > x+1 $	$3 x > x+1 $	$3 x > x+1 $
\checkmark	\times	\checkmark

solutions: $x < -\frac{1}{4}$ or $x > \frac{1}{2}$. \checkmark (4)

6. (i) $f(x) = 2\ln(x-3)$



(ii) $f(x) = |2\ln(x-3)|$



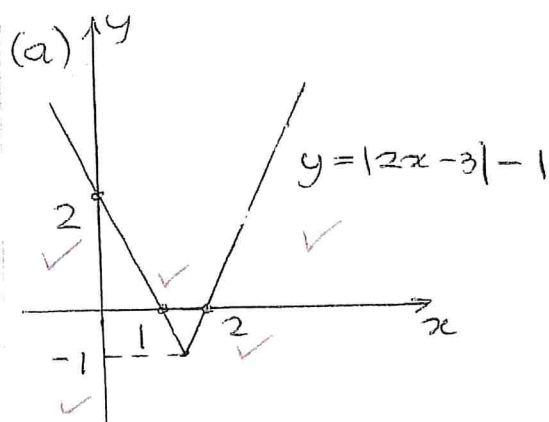
7. (a) $\frac{360}{2} = 180^\circ$ or π rad ✓

(b) $P(180^\circ, 0)$ ✓

(c) $Q(45^\circ, 3)$ or $(\frac{\pi}{4}, 3)$ ✓

(d) → horizontal stretch $sf = \frac{1}{2}$ ✓
 ↑ vertical stretch, $sf = 3$. ✓

8.



(6)

(5)

(b) $f(x) = x$

$|2x-3|-1 = x$

$2x-3-1 = x$

$x = 4$ ✓

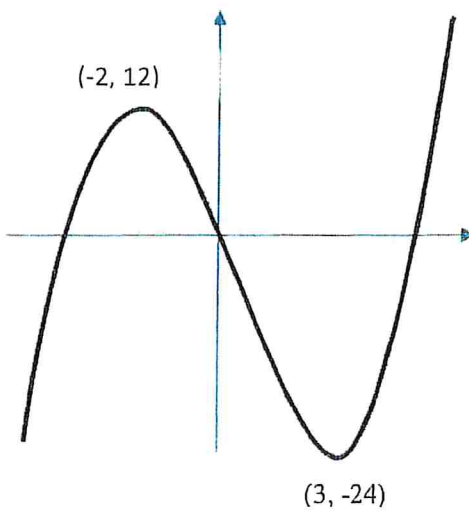
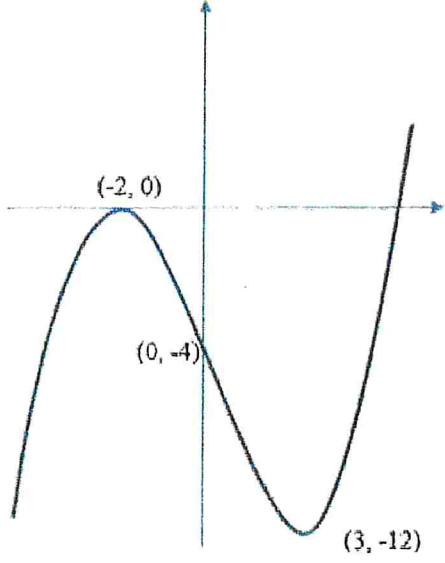
or $-(2x-3)-1 = x$ ✓

$-2x+3-1 = x$

$2 = 3x$

$\frac{2}{3} = x$ ✓

(3)

9(a)		<p>B1</p> <p>B1</p> <p>(2)</p>
(b)		<p>M1</p> <p>A1</p> <p>A1</p> <p>(3)</p>
		(5 marks)

TOTAL : 47