

Functions

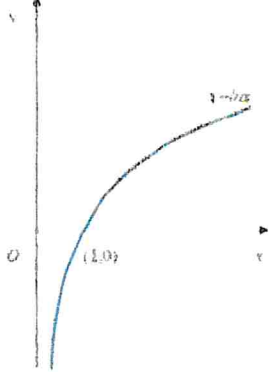
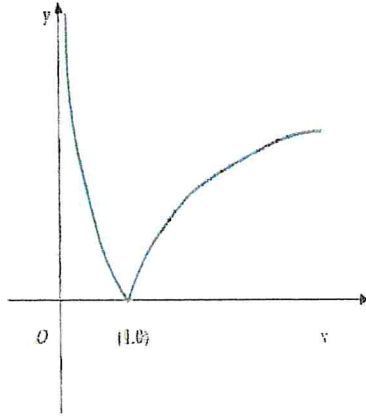
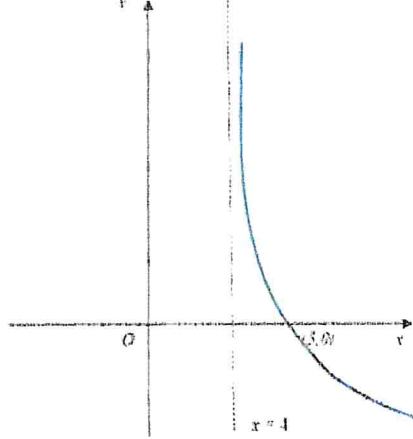
SOLUTIONS

Section 1

1.	$x^2 - 9 = (x+3)(x-3)$	
	$\frac{4x}{x^2 - 9} - \frac{2}{x+3} = \frac{4x - 2(x-3)}{(x+3)(x-3)}$	
	$= \frac{2x+6}{(x+3)(x-3)}$	
	$= \frac{2\cancel{(x+3)}}{\cancel{(x+3)}(x-3)}$	
	$= \frac{2}{x-3}$	

2.	$ \begin{array}{r} 3x^2 - 2x + 7 \\ x^2(+0x) - 4 \overline{) 3x^4 - 2x^3 - 5x^2 + (0x) - 4} \\ \underline{3x^4 + 0x^3 - 12x^2} \\ - 2x^3 + 7x^2 + 0x \\ \underline{- 2x^3 + 0x^2 + 8x} \\ 7x^2 - 8x - 4 \\ \underline{7x^2 + 0x - 28} \\ - 8x + 24 \end{array} $	
		$a = 3$

	$ \begin{array}{r} 3x^2 - 2x \dots\dots \\ x^2(+0x) - 4 \overline{) 3x^4 - 2x^3 - 5x^2 + (0x) - 4} \\ \underline{3x^4 + 0x^3 - 12x^2} \\ - 2x^3 + \dots\dots\dots \\ \underline{- 2x^3 + \dots\dots\dots} \end{array} $ <p>Long division as far as</p>	
	Two of $b = -2$ $c = 7$ $d = -8$ $e = 24$	
	All four of $b = -2$ $c = 7$ $d = -8$ $e = 24$	

<p>3.</p>		<p>In graph crossing x axis at (1,0) and asymptote at $x = 0$</p>	<p>B1</p>
		<p>Shape including cusp Touches or crosses the x axis at (1,0) Asymptote given as $x=0$</p>	<p>B1ft B1ft B1</p>
		<p>Shape Crosses at (5, 0) Asymptote given as $x = 4$</p>	<p>B1 B1ft B1</p>

Section 2

① a) $D: x \in \mathbb{R}$ ✓
 $R: y \geq -2, y \in \mathbb{R}$ ✓

b) $x \in \mathbb{R}$ domain ✓
 $y \in \mathbb{R}$ range ✓

c) $D: x \geq 4, x \in \mathbb{R}$ ✓
 $R: y \geq 0, y \in \mathbb{R}$ ✓

(6)

② $f(x) = 4x - 5, x \in \mathbb{R}$

a) $f(3) = 4 \times 3 - 5 = 7$ ✓

b) $f(-2) = 4(-2) - 5 = -13$ ✓

c) $f\left(\frac{1}{4}\right) = 4 \times \frac{1}{4} - 5 = -4$ ✓

d) $y = 4x - 5$

$y + 5 = 4x$

$\frac{y+5}{4} = x \Rightarrow x = \frac{y+5}{4}, f^{-1}(x) = \frac{x+5}{4}$ ✓

Domain of f^{-1} :

$x \in \mathbb{R}$ ✓

(5)

③ $f(x) = x^3, x \in \mathbb{R}$ and $g(x) = 4x - 1, x \in \mathbb{R}$

a) $fg(x) = f(4x - 1) = (4x - 1)^3$ ✓

b) $gf(x) = g(x^3) = 4x^3 - 1$ ✓

c) $gg(x) = g(4x - 1) = 4(4x - 1) - 1 = 16x - 4 - 1$

$= 16x - 5$ ✓

(4)

④ a) $R: y \geq 3, y \in \mathbb{R}$ ✓

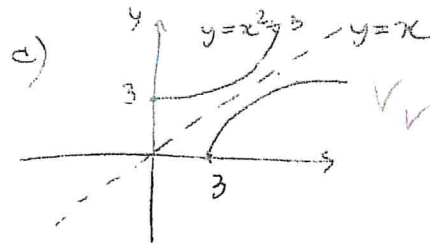
b) $y = x^2 + 3$

$y - 3 = x^2$

$x^2 = y - 3$

$x = \sqrt{y - 3}$

$f^{-1}(x) = \sqrt{x - 3}$ ✓, $D: x \geq 3, x \in \mathbb{R}$



(6)

d) They are a mirror image of each other in the line $y = x$. ✓

$$\textcircled{5} \quad f(x) = \frac{x}{x+5} - 3, \quad x \neq -5$$

$$y = \frac{x}{x+5} - 3$$

$$y+3 = \frac{x}{x+5}$$

$$(y+3)(x+5) = x \quad \checkmark$$

$$yx + 5y + 3x + 15 = x$$

$$yx + 5y + 2x + 15 = 0$$

$$x(y+2) = -(5y+15)$$

$$x = -\frac{5y+15}{y+2} \quad \checkmark$$

(4)

$$f^{-1}(x) = -\frac{5x+15}{x+2} \quad \text{D: } x \in \mathbb{R} \quad x \neq -2$$

$$\textcircled{6} \quad f(x) = 5x+6 \quad g(x) = \frac{2}{x} \quad x \neq 0$$

$$\text{a) } fg = f\left(\frac{2}{x}\right) = 5 \times \frac{2}{x} + 6 = \frac{10}{x} + 6 \quad \checkmark$$

$$gg = g(g(x)) = g\left(\frac{2}{x}\right) = \frac{2}{\frac{2}{x}} = 2 \div \frac{2}{x} = 2 \times \frac{x}{2} = \frac{2x}{2} = x$$

$$\text{b) } (fg)^{-1} \quad fg = \frac{10}{x} + 6$$

$$y = \frac{10}{x} + 6 \quad \times x$$

$$yx = 10 + 6x \quad \checkmark$$

$$yx - 6x = 10$$

$$x(y-6) = 10$$

$$x = \frac{10}{y-6}$$

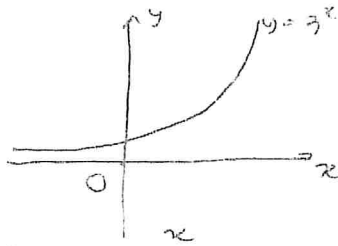
$$(fg)^{-1} = \frac{10}{x-6}$$

Domain of $(fg)^{-1}$:

$$x \in \mathbb{R} \quad x \neq 6 \quad \checkmark$$

(6)

(7) $f(x) = 3^x, x \in \mathbb{R}$



(i) $\mathbb{R} : y > 0, y \in \mathbb{R} \checkmark$

(ii) $f(-2) = 3^{-2} = \frac{1}{3^2} = \frac{1}{9} \checkmark$

(iii) one-one \checkmark

(iv) $y = 3^x$ take \log_3 of both sides

$$\log_3 y = \log_3 3^x$$

$$\log_3 y = x \log_3 3$$

$$x \log_3 3 = \log_3 y \checkmark$$

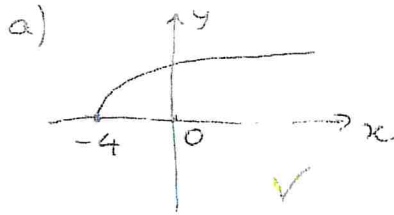
$$x = \frac{\log_3 y}{\log_3 3} = \log_3 y$$

$$f^{-1}(x) = \log_3 x$$

D: $x > 0, x \in \mathbb{R}$

(5)

(8) $f(x) = \sqrt{x+4}, x \geq -4, x \in \mathbb{R}$



b) $\mathbb{R} : y \geq 0, y \in \mathbb{R} \checkmark$

c) $y = \sqrt{x+4}$ square both sides

$$y^2 = x+4$$

$$y^2 - 4 = x$$

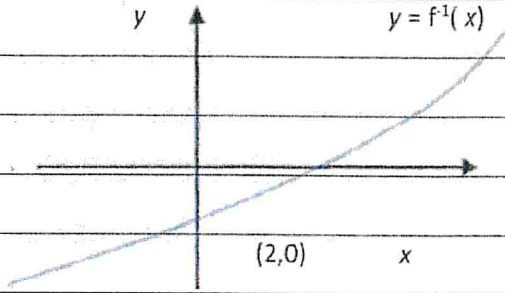
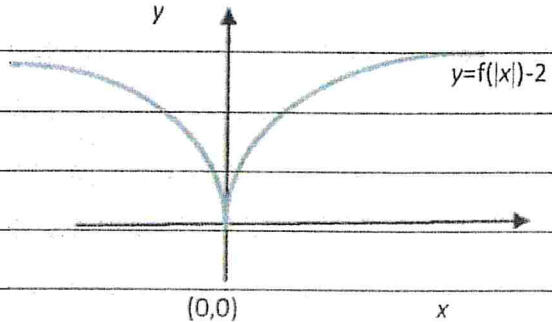
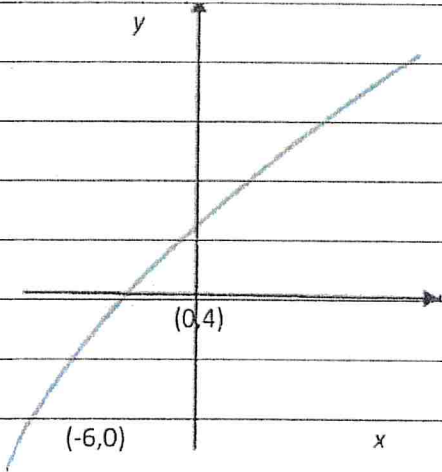
$$x = y^2 - 4 \checkmark$$

$$f^{-1}(x) = x^2 - 4 \checkmark$$

D: $x \geq 0, x \in \mathbb{R} \checkmark$

(5)

9(a)	$fg(x) = \frac{28}{x-2} - 1$	M1
	$\left(= \frac{30-x}{x-2} \right)$	
	Sets $fg(x) = x \Rightarrow \frac{28}{x-2} - 1 = x$	
	$\Rightarrow 28 = (x+1)(x-2)$	M1
	$\Rightarrow x^2 - x - 30 = 0$	
	$\Rightarrow (x-6)(x+5) = 0$	dM1
	$\Rightarrow x = 6, x = -5$	A1
		(4)
(b)	$a = 6$	B1ft
		(1)
		(5 marks)

10(a)	$ff(-3) = f(0), = 2$	M1 A1
		(2)
(b)		
	Shape	B1
	(0,-3) (2,0)	(0,-3) and (2,0)
		B1
		(2)
(c)		
	Shape	B1
	(0,0)	(0,0)
		B1
		(2)
(d)		
	Shape	B1
		(-6,0) or (0,4)
		B1
		(-6,0) and (0,4)
		B1
		(3)
		(9 marks)

TOTAL: 55 marks