PURE 9 - SOLUTIONS

Section 1

1 a) $\cos x = 0.9$

  $x = \underline{25.8°, 334°}$

b) $\sin x = 0.41$

  $x = \underline{24.2, 156°}$

c) $\tan x = 0.74$

  $x = \underline{36.5°, 217°}$

2 a) $-10 \le x - 10 \le 350$

  $x - 10 = \sin^{-1}(0.5) = 30, 150$

  $x = \underline{40°, 160°}$

b) $0 \le 2x \le 720$

  $2x = \cos^{-1}(0.5)$

  $= 60, 300, 420, 660$

  $x = \underline{30°, 150°, 210°, 330°}$

3) a) $\tan(x-15) = 1.85$

  $x - 15 = 61.6, -118.4$

  $x = \underline{76.6°, -103.4°}$

b) $\cos 3x = \frac{1}{2}$

  $3x = 60, 300, 420, -60, -300, -4$

  $x = \underline{-140°, -100°, -20°, 20°, 100°, 140°}$

c) $2x + 30 = \tan^{-1}(-1) = -45, 135, 315, -255$

  $x = \underline{-37.5°, 52.5°, 142.5°, -127.5°}$

4) a) $\cos x (2\cos x + 1) = 0$

  $\cos x = 0$ OR $\cos x = -\frac{1}{2}$

  $x = \underline{90°, 270°, 120°, 240°}$

b) $\tan x = \dfrac{-3 \pm \sqrt{13}}{2}$ $x = \underline{106.8°, 286.8°, 16.8°, 196.8°}$

5 a) $\sin^2 x + \cos^2 x = 1$

b) $\tan x = \dfrac{\sin x}{\cos x}$

6 a) $\cos x = \dfrac{3 \sin x}{\cos x}$

  $\cos^2 x = 3 \sin x$

  $1 - \sin^2 x = 3 \sin x$

  $\sin^2 x + 3 \sin x - 1 = 0$

  $\sin x = -\dfrac{3 \pm \sqrt{13}}{2}$ no solution for $\sin x = \dfrac{-3 - \sqrt{13}}{2}$

  $x = \sin^{-1}\left(\dfrac{-3 + \sqrt{13}}{2}\right) = \underline{17.6°, 162.4°}$

6b) $2(1-\cos^2 x) + 3\cos x - 3 = 0$

$2 - 2\cos^2 x + 3\cos x - 3 = 0$

$2\cos^2 x - 3 \qquad +1 = 0$

$(2\cos x - 1)(\cos x - 1) = 0$

$\cos x = \frac{1}{2} \qquad\qquad \cos x = 1$

$x = 60°, 300° \quad, 0°, 360°$

c) $\dfrac{3\sin x \times \sin x}{\cos x} = 8 \quad \Rightarrow \quad 3\sin^2 x = 8\cos x$

$\Rightarrow 3(1-\cos^2 x) = 8\cos x$

$\Rightarrow 3 - 3\cos^2 x = 8\cos x$

$3\cos^2 x + 8\cos x - 3 = 0$
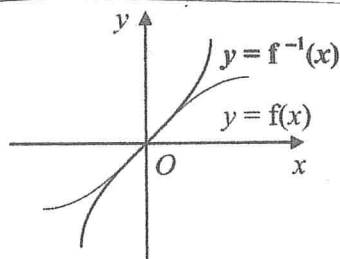
$(3\cos x - 1)(\cos x + 3) = 0$

$\cos x = \frac{1}{3} \qquad\qquad \rightarrow$ no solution

$x = 70.5°, 289.5°$

Section 2

1)a) $-1 \le f(x) \le 1$ ✓

b) $f^{-1}(x) = \arcsin x, \; x \in \mathbb{R}, \; -1 \le x \le 1$ ✓

c)

✓ (2) For $\sin x$

✓ (2) For $\arcsin x$

2a) $x = \cos 2 = -0.416$ ✓

b) $3x = \tan 0.96 = 1.42836 \Rightarrow x = 0.476$ ✓

c) $\arccos 2x = 3 \Rightarrow 2x = \cos 3 = -0.98999$ ✓

$x = -0.495$ ✓

3 a) $f(-\tfrac{1}{2}) = \tfrac{2\pi}{3} - \tfrac{\pi}{3} = \tfrac{\pi}{3}$ ✓

b) $\arccos x = \tfrac{\pi}{3}$ ✓ $\Rightarrow x = \cos\tfrac{\pi}{3} = \tfrac{1}{2}$ ✓

c) $y = \arccos x - \tfrac{\pi}{3}$

$x = \text{arcos}\, y - \tfrac{\pi}{3}$ ✓
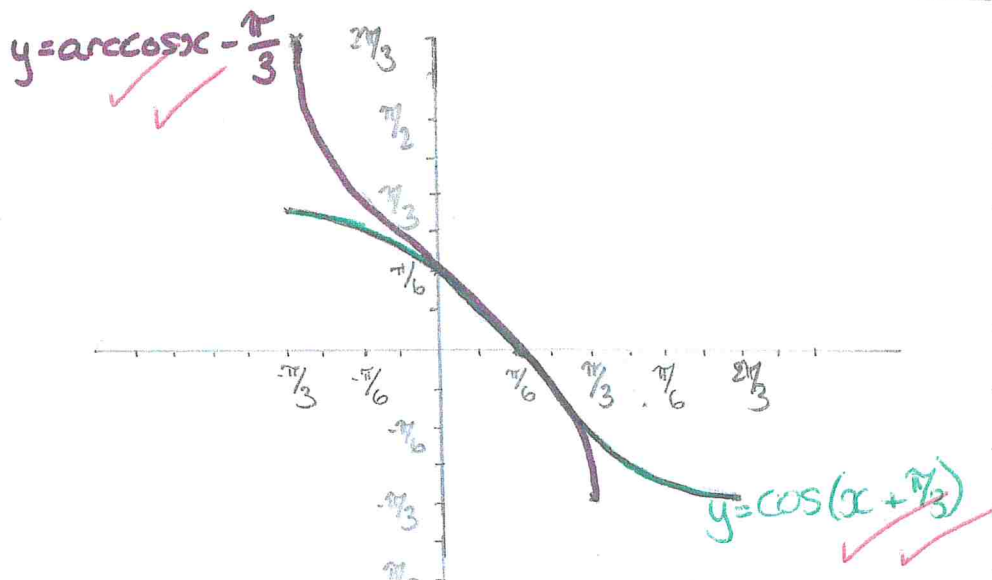
$x + \tfrac{\pi}{3} = \text{arcos}\, y$ ✓

$y = \cos(x + \tfrac{\pi}{3})$ ✓

$\rightarrow f^{-1}(x) = \cos(x + \tfrac{\pi}{3})$ ✓

$x \in \mathbb{R}$

$-\tfrac{\pi}{3} \leq x \leq \tfrac{2\pi}{3}$ ✓

d)

$y = \arccos x - \tfrac{\pi}{3}$ ✓

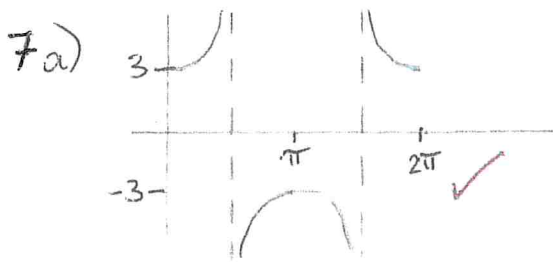$y = \cos(x + \tfrac{\pi}{3})$ ✓

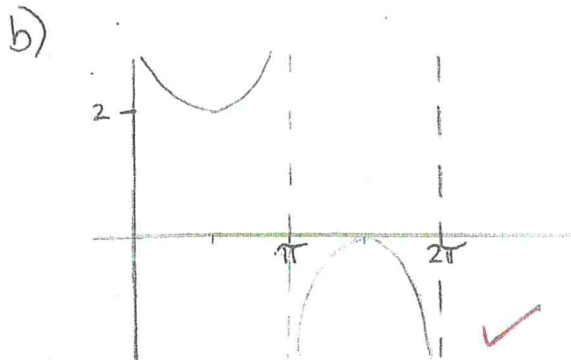4a) 1.09 ✓   b) −11.47 ✓   c) 0.19 ✓

5a) $\sqrt{2}$ ✓   b) −1 ✓   c) −2 ✓

6a) (0,1), (180,−1), (360,1), (540,−1), (720,1) ✓

b) $x = 90$, $x = 270$, $x = 450$, $x = 630$ ✓

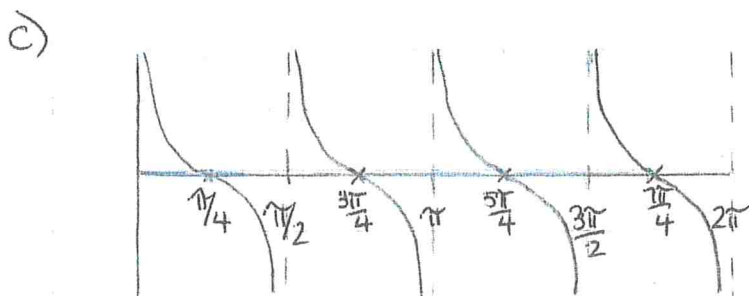**7a)**

turning points :
$(0,3), (\pi,-3), (2\pi,3)$ ✓

Asymptotes :
$x = \frac{\pi}{2}$ and $x = \frac{3\pi}{2}$ ✓

b)

turning points :
$(\frac{\pi}{2}, 2)$ $(\frac{3\pi}{2}, 0)$ ✓

Asymptotes
$x = 0, x = \pi, x = 2\pi$ ✓

c)

✓ for shape
✓ for axes labelled.

Asymptotes at $x = \frac{\pi}{2}, x = \pi, x = \frac{3\pi}{2}, x = 2\pi$ ✓

8 a) $\dfrac{1}{\cos\theta} = 1.8 \Rightarrow \cos\theta = 5/9$ ✓

$\theta = 0.982^c, 5.30^c$ ✓

b) $\dfrac{1}{\sin(\theta + \frac{\pi}{6})} = 2 \Rightarrow \sin(\theta + \frac{\pi}{6}) = \frac{1}{2}$ ✓

$\theta + \frac{\pi}{6} = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}$ ✓

$\theta = 0, \frac{2\pi}{3}, 2\pi$ ✓

c) $\dfrac{1}{\sin^2\theta} = 4 \Rightarrow \sin^2\theta = \frac{1}{4}$ ✓

$\sin\theta = \pm\frac{1}{2}$ ✓

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$ ✓

d) $(\sec\theta - 3)(\sec\theta + 1) = 0$ ✓

$\sec\theta = 3$ $\sec\theta = -1$ ✓

$\cos\theta = \frac{1}{3}$ $\cos\theta = -1$ ✓

$\theta = 1.23, 5.05, \pi$ ✓

9) a) LHS: $\sec x - \cos x \equiv \dfrac{1}{\cos x} - \cos x$ ✓

$$\equiv \dfrac{-\cos^2 x}{\cos x}$$ ✓

$$\equiv \dfrac{\sin^2 x}{\cos x}$$ ✓

$$\equiv \sin x \times \dfrac{\sin x}{\cos x}$$

$$\equiv \sin x \tan x \quad \text{(RHS)}$$ ✓

b) LHS $(1 + \cos x)(\csc x - \cot x)$

$$\equiv \csc x - \cot x + \cos x \csc x - \cos x \cot x$$

$$\equiv \csc x - \cot x + \cos x \times \dfrac{1}{\sin x} - \cos x \times \dfrac{\cos x}{\sin x}$$ ✓

$$\equiv \dfrac{1}{\sin x} - \cot x + \cot x - \dfrac{\cos^2 x}{\sin x}$$ ✓

$$\equiv \dfrac{1 - \cos^2 x}{\sin x}$$ ✓

$$\equiv \dfrac{\sin^2 x}{\sin x}$$ ✓

$$\equiv \sin x \quad \text{(RHS)}$$