Double Angles and $R\sin(x+\alpha)$ SOLUTIONS

SECTION 1

1) $y = x^{1/2} + 4x^{-1/2} + 4$

$\frac{dy}{dx} = \frac{1}{2}x^{-1/2} - 2x^{-3/2}$. when $x = 8$ $\frac{dy}{dx} = \frac{1}{16}\sqrt{2}$

2) a) $90 = 20 + Ae^{0} \Rightarrow A = 70$

b) $t = 5\ \theta = 55 \Rightarrow 55 = 20 + 70e^{-k\times 5}$

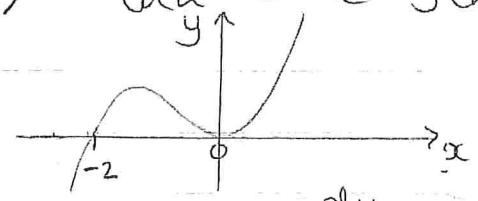
$\Rightarrow e^{-5k} = 35/70$ $\quad -5k = \ln(35/70)$

$\Rightarrow -5k = \ln 1/2\ (= \ln 2^{-1}) = -\ln 2 \Rightarrow k = \frac{1}{5}\ln 2$.

3) a) $y = x^3 + 2x^2$

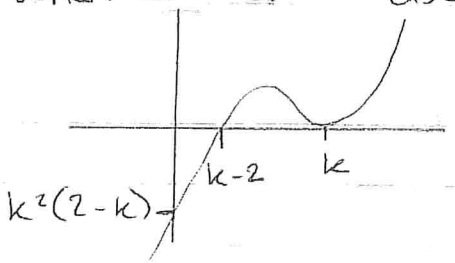
$\frac{dy}{dx} = 3x^2 + 4x$

b)



c) when $x = -2$ $dy/dx = 4$, when $x = 0$ $dy/dx = 0$

d)



$y = (x-k)^2((x-k)+2)$

translation k to right.

$x = 0$ $y = (-k)^2(2-k) = k^2(2-k)$

4a) $\log_5 10 = x$ $\quad x = 1.43$

b) $3^{-1} = x - 2 \Rightarrow x - 2 = 1/3$ $\quad x = 2\,1/3$

5) a) $\log_2 (x+15)^2 - \log_2 x = 6$

$\Rightarrow \log_2\left(\frac{(x+15)^2}{x}\right) = 6 \Rightarrow \frac{(x+15)^2}{x} = 2^6 = 64$

$x^2 + 30x + 225 = 64x \Rightarrow x^2 - 34x + 225 = 0$

b) $(x-25)(x-9) = 0$ $\quad x = 9$ or 25

6) $\int 2x^5 - \frac{1}{4}x^{-3} - 5\,dx = \frac{1}{3}x^6 + \frac{1}{8}x^{-2} - 5x + C$

Section 2.

1a) $3\cos\theta + 4\sin\theta = R\cos(\theta - \alpha) = R\cos\theta\cos\alpha + R\sin\theta\sin\alpha$

$3 = R\cos\alpha$ ✓ $\quad R^2 = 3^2 + 4^2 \quad R = 5$ ✓

$4 = R\sin\alpha \quad \tan\alpha = 4/3 \quad \alpha = 53.1°$ ✓

b) $5\cos(\theta - 53.1) = 1 \Rightarrow \theta - 53.1 = \cos^{-1}(1/5)$ ✓

$= 78.5, 281.5$ ✓ ✓

$\theta = 131.6°, 334.6°$

(7)

c) min $= -5$. ✓

2a) (LHS)

$$\frac{\cos 2x}{\cos x - \sin x} \equiv \frac{\cos^2 x - \sin^2 x}{\cos x - \sin x} \equiv \frac{(\cos x + \sin x)(\cos x - \sin x)}{\cos x - \sin x}$$

$\equiv \cos x + \sin x$ (RHS)

b) $1\cos x + 1\sin x = R\sin(x + \alpha)$

$= R\sin x\cos\alpha + R\cos x\sin\alpha$

$1 = R\cos\alpha \quad 1 = R\sin\alpha$ ✓

(9)

$R^2 = 1^2 + 1^2 \Rightarrow R = \sqrt{2}$ ✓ $\quad \tan\alpha = 1 \Rightarrow \alpha = 45°$ ✓

$\cos x + \sin x = \sqrt{2}\sin(x + 45)$

$$\frac{\cos 2x}{\cos x - \sin x} = \frac{1}{2} \Rightarrow \cos x + \sin x = \frac{1}{2}$$

$\Rightarrow \sqrt{2}\sin(x + 45) = 1/2$ ✓

$\Rightarrow x + 45 = \sin^{-1}(1/2\sqrt{2})$

$= 20.7, 159.3, 380.7$

$x = 114.3°, 335.7°$ ✓ ✓

3. LHS: $\dfrac{2\tan x}{1 + \tan^2 x} = \dfrac{2\sin x / \cos x}{\sec^2 x}$ ✓ ✓

(4)

$$\equiv \frac{2\sin x}{\cos x} \times \cos^2 x$$ ✓

$\equiv 2\sin x\cos x = \sin 2x$. ✓

4a) $1\sin x + \sqrt{3}\cos x = R\sin(x + \alpha)$

$= R\sin x\cos\alpha + R\cos x\sin\alpha$

$1 = R\cos\alpha \quad \sqrt{3} = R\sin\alpha$ ✓

$R = \sqrt{(\sqrt{3})^2 + 1^2} = 2$ ✓ $\quad \tan\alpha = \sqrt{3} \quad \alpha = 60°$ ✓

$\sin x + \sqrt{3}\cos x = 2\sin(x + 60)$

b) $\sec x + \sqrt{3}\operatorname{cosec} x = 4 \Rightarrow \frac{1}{\cos x} + \frac{\sqrt{3}}{\sin x} = 4$ ✓

(7)

$\Rightarrow \sin x + \sqrt{3}\cos x = 4\sin x \cos x$ ✓

$\Rightarrow \sin x + \sqrt{3}\cos x = 2\sin 2x$ ✓

c) $\sec x + \sqrt{3}\operatorname{cosec} x = 4 \Rightarrow \sin x + \sqrt{3}\cos x = 2\sin 2x$

$\Rightarrow 2\sin(x+60) = 2\sin 2x$

$\Rightarrow \sin 2x - \sin(x+60) = 0$ ✓

5) $2\cos 2\theta = 1 - 2\sin\theta \Rightarrow 2(1-2\sin^2\theta) = 1 - 2\sin\theta$ ✓

$\Rightarrow 4\sin^2\theta - 2\sin\theta - 1 = 0$ ✓

$\Rightarrow \sin\theta = \frac{1+\sqrt{5}}{4}$ or $\frac{1-\sqrt{5}}{4}$ ✓

(6)

$\Rightarrow \theta = (-18), 54, 126, 198, 342$ ✓

6)a) $6\cos\theta + 8\sin\theta = R\cos(\theta - \alpha)$

$= R\cos\theta\cos\alpha + R\sin\theta\sin\alpha$

$6 = R\cos\alpha \qquad 8 = R\sin\alpha$ ✓

$R = \sqrt{6^2+8^2} = 10$ ✓ $\qquad \tan\alpha = 8/6 \Rightarrow \alpha = 0.927$ ✓

(8)

$6\cos\theta + 8\sin\theta = 10\cos(\theta - 0.927)$ ✓

b) $p(\theta) = \frac{4}{12 + 10\cos(\theta - 0.927)}$

(i) max: $p(\theta) = \frac{4}{12-10}$ ✓ $= 2$ ✓

(ii) $\cos(\theta - 0.927) = -1 \Rightarrow \theta - 0.927 = \pi$ ✓

$\theta = 4.07$ ✓

7(i) $(\sin 22.5 + \cos 22.5)^2 = \sin^2 22.5 + 2\sin 22.5\cos 22.5 + \cos^2 22.5$ ✓

$= 1 + \sin 45$ ✓ $= 1 + \sqrt{2}/2$ ✓

* $\sin^2 22.5 + \cos^2 22.5 = 1$ ✓

* $2\sin 22.5\cos 22.5 = \sin(2\times 22.5) = \sin 45$ ✓

(ii)a) $\cos 2\theta + \sin\theta = 1$

$(1 - 2\sin^2\theta) + \sin\theta = 1$ ✓

$\Rightarrow \sin\theta - 2\sin^2\theta = 1$

$\Rightarrow 2\sin^2\theta - \sin\theta = 1 \qquad k = 2$ ✓

b) $\cos 2\theta + \sin\theta = 1 \Rightarrow 2\sin^2\theta - \sin\theta = 0$ ✓

$\Rightarrow \sin\theta(2\sin\theta - 1) = 0$ ✓

(11) $\Rightarrow \sin\theta = 0 \quad \sin\theta = \frac{1}{2}$ ✓

$\Rightarrow \theta = 0, 30, 150, 180$ ✓✓

8a) $24\sin\theta + 7\cos\theta = R\cos(\theta - \alpha)$

$= R\cos\theta\cos\alpha + R\sin\theta\sin\alpha$

$24 = R\sin\alpha \qquad 7 = R\cos\alpha$

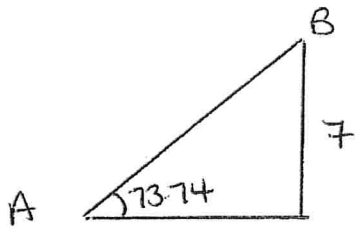
$R = \sqrt{24^2 + 7^2} = 25$ ✓

$\tan\alpha = 24/7$ ✓ $\quad \alpha = 73.74$ ✓

b) $V = \dfrac{21}{25\cos(\theta - 73.74)}$ ✓ $\quad V_{min} = \dfrac{21}{25}$ ✓

c) V_{min} occurs when $\cos(\theta - 73.74) = 1$

(8) ie $\theta = 73.74$. ✓



$\sin 73.74 = \dfrac{7}{AB}$ ✓

$AB = 7.29\text{m}$. ✓

TOTAL 60