- SECTION 1

1 a) $6x$ b) $3x^2$ c) $5x^4 + 12x^2 + 2$

d) 0 e) 2 f) $4x^3 + 4$

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2 a) $\frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$ b) $-12x^{-3} = \frac{-12}{x^3}$ c) $1 - \frac{1}{x^2}$

d) $\frac{2}{3}x^{-1/3} + \frac{5}{3}x^{2/3}$ e) $2x + \frac{16}{x^3}$

f) $y = 2x^{5/2} + 3x^{1/2}$ $\therefore$ $\underline{\underline{\frac{dy}{dx} = 5x^{3/2} + \frac{3}{2}x^{-1/2}}}$

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3. $f(x) = x^2$ $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$$\therefore f'(x) = \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h}$$

$$= \lim_{h \to 0} \frac{x^2 + h^2 + 2hx - x^2}{h}$$

$$= \lim_{h \to 0} \frac{h^2 + 2hx}{h}$$

$$= \lim_{h \to 0} h + 2x$$

$$= \underline{\underline{2x}}$$

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4. $y = 2x^3 + 6x + 10$

$\therefore \frac{dy}{dx} = 6x^2 + 6$

at $x = -1$ $\quad \frac{dy}{dx} = 6(-1)^2 + 6 = 12$

$\therefore y - 2 = 12(x+1) \quad \therefore \underline{\underline{y = 12x + 14}}$

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5. $y = x^2 + 1 \quad \therefore \frac{dy}{dx} = 2x$

at $(2,5)$ $\quad \frac{dy}{dx} = 2 \times 2 = 4.$

$\therefore$ equation of tangent is $y - 5 = 4(x-2) \therefore y = 4x - 3$

at $(1,2)$ $\quad \frac{dy}{dx} = 2 \times 1 = 2 \quad \therefore$ gradient of normal $= -\frac{1}{2}$

$\therefore$ equation of normal is $y - 2 = -\frac{1}{2}(x-1) \therefore y = -\frac{1}{2}x + \frac{5}{2}$

at point of intersection $4x - 3 = -\frac{1}{2}x + \frac{5}{2}$

$\therefore x = \frac{11}{9}$

$\therefore y = 4 \times \frac{11}{9} - 3 = \frac{17}{9}$

$\therefore$ $\underline{\underline{\text{coordinates are } \left(\frac{11}{9}, \frac{17}{9}\right)}}$

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6. $\sqrt{75} - \sqrt{12} = 5\sqrt{3} - 2\sqrt{3} = 3\sqrt{3}$ $\quad$ $\underline{\underline{a = 3 \quad b = 3}}$

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7. $2 + 0.8x - 0.04x^2 = 2 - 0.04(x^2 - 20x)$

$= 2 - 0.04\left((x-10)^2 - 100\right) = -0.04(x-10)^2 + 6$

$\therefore$ $\underline{\underline{A = 6 \quad B = 0.04 \quad C = -10}}$

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8. $b^2 - 4ac = 0$ $\quad a = 5 \quad b = 8 \quad c = s$

$\therefore 8^2 - 4 \times 5 \times s = 64 - 4s^2 = 0$ $\quad \therefore$ since $s > 0$, $\underline{\underline{s = 4}}$

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9. $y = x + k$ $\quad \therefore x^2 + (x+k)^2 = 4$

$\therefore 2x^2 + 2xk + k^2 = 4$

$\therefore 2x^2 + 2xk + (k^2 - 4) = 0$

one pair of solutions $\therefore$ discriminant $= 0$

$(2k)^2 - 4 \times 2(k^2 - 4) = 0$

$\therefore -4k^2 + 32 = 0$

$\therefore k^2 = 8$

$\therefore \underline{\underline{k = \pm 2\sqrt{2}}}$

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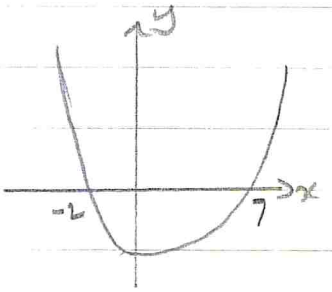
10. $y = x^2 - 5x - 14 = (x-7)(x+2)$

$\therefore$ critical values of $x^2 - 5x - 14 > 0$ are $x = 7, -2$



Hence $x < -2$ or $x > 7$

$$\{x : x < -2\} \cup \{x : x > 7\}$$

Section 2

1\. a) $-2\sin x$ b) $4\cos 4x$ c) $\frac{5\pi}{3}\cos\left(\frac{\pi x}{3}\right)$

d) $6\cos 2x - 5\sin x$ e) $9x^2 + 2\cos x$

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2\. $f(x) = \cos 3x$ $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

$$f'(x) = \lim_{h \to 0} \frac{\cos 3(x+h) - \cos 3x}{h}$$

$$= \lim_{h \to 0} \frac{\cos 3x \cos 3h - \sin 3x \sin 3h - \cos 3x}{h}$$

$$= \lim_{h \to 0} \cos 3x \left(\frac{\cos 3h - 1}{h}\right) - \sin 3x \left(\frac{\sin 3h}{h}\right)$$

$$= \lim_{h \to 0} 3\cos 3x \left(\frac{\cos 3h - 1}{3h}\right) - 3\sin 3x \left(\frac{\sin 3h}{3h}\right)$$

We have $\dfrac{\sin h}{h} \approx 1$ $\therefore$ $\dfrac{\sin 3h}{3h} \approx 1$ for small h

and $\dfrac{\cos h - 1}{h} \approx -\dfrac{h}{2}$ $\therefore$ $\dfrac{\cos 3h - 1}{3h} \approx -\dfrac{3h}{2}$ for small h

$$\therefore f'(x) = \lim_{h \to 0} 3\cos 3x \left(-\frac{3h}{2}\right) - 3\sin 3x$$

$$\therefore \underline{\underline{f'(x) = -3\sin 3x}}$$

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3. $y = x + \cos x \quad \therefore \quad \frac{dy}{dx} = 1 - \sin x$

at $x = \frac{\pi}{6} \quad \frac{dy}{dx} = 1 - \sin\frac{\pi}{6} = 1 - \frac{1}{2} = \frac{1}{2}$

$y = \frac{\pi}{6} + \cos\frac{\pi}{6} = \frac{\pi}{6} + \frac{\sqrt{3}}{2}$

$\therefore \quad y - \left(\frac{\pi}{6} + \frac{\sqrt{3}}{2}\right) = \frac{1}{2}\left(x - \frac{\pi}{6}\right)$

$\therefore \quad y = \frac{1}{2}x + \frac{\pi}{12} + \frac{\sqrt{3}}{2}$

4

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4. a) $3e^{3x}$ b) $-2e^{-2x} + \frac{1}{x}$

c) $y = 5^x \therefore \ln y = x\ln 5 \therefore y = e^{x\ln 5}$

$\therefore \frac{dy}{dx} = \ln 5 \, e^{x\ln 5} = \ln 5 \, e^{\ln y} = y\ln 5 = 5^x \ln 5$

d) $f(x) = \ln 4 + \ln x^5 = \ln 4 + 5\ln x$

$\therefore f'(x) = \frac{5}{x}$

e) $y = 2^{3x-1} \therefore \ln y = (3x-1)\ln 2 \therefore y = e^{(3x-1)\ln 2}$

$\therefore \frac{dy}{dx} = 3\ln 2 \, e^{(3x-1)\ln 2} = 3\ln 2 \times 2^{3x-1}$

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5. $y = \ln x + 3/x \quad \therefore \quad \frac{dy}{dx} = \frac{1}{x} - \frac{3}{x^2}$

at $x = 1 \quad \frac{dy}{dx} = -2 \quad \therefore$ gradient of normal $= \frac{1}{2}$

$y = \ln 1 + 3/1 = 3$

$\therefore \quad y - 3 = \frac{1}{2}(x - 1)$

$\therefore \quad \underline{\underline{y = \frac{1}{2}x + \frac{5}{2}}}$

(5)

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6 a) $\frac{dy}{dx} = 5(3+2x)^4 \times 2 = \underline{\underline{10(3+2x)^4}}$

b) $f'(x) = -4(3-2x)^{-5} \times -2 = \underline{\underline{8(3-2x)^{-5}}}$

c) $\frac{dy}{dx} = 3(2+3x^2)^2 \times 6x = \underline{\underline{18x(2+3x^2)^2}}$

d) $f'(x) = 5(x^2+3x+1)^4 \times (2x+3) = \underline{\underline{5(2x+3)(x^2+3x+1)^4}}$

e) $\frac{dy}{dx} = \frac{1}{2} \times 5(x^2-1)^{-\frac{1}{2}} \times 2x = \underline{\underline{5x(x^2-1)^{-\frac{1}{2}}}}$

(15)

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7 $\quad y = (e^x + \ln x)^2 \quad \therefore \quad \frac{dy}{dx} = 2(e^x + \ln x)(e^x + 1/x)$

at $x = 1 \quad \frac{dy}{dx} = 2(e^1 + \ln 1)(e^1 + 1/1) = 2e(e+1)$

$y = (e^1 + \ln 1)^2 = e^2$

(5)

$\therefore \quad y - e^2 = 2e(e+1)(x-1) = \underline{\underline{2e(e+1)x - e(e+2)}}$

8. $y = 4 - e^{x}$ $\therefore \frac{dy}{dx} = -e^{x}$

a) at P $x = 0$ $\therefore \frac{dy}{dx} = -1$ $\therefore$ gradient of normal $= 1$

at P $y = 4 - e^{0} = 3$ $\therefore$ P is $(0, 3)$

$\therefore$ equation of normal at P is $y - 3 = x$

$\therefore$ $\underline{y = x + 3}$

b) at Q $y = 0$ $\therefore$ $4 - e^{x} = 0$ $\therefore$ $x = \ln 4$
$\therefore$ Q is $(\ln 4, 0)$

$\frac{dy}{dx} = -e^{\ln 4} = -4$

$\therefore$ equation of tangent at Q is $y = -4(x - \ln 4)$

$\ln 4 = \ln 2^{2} = 2\ln 2$ $\therefore$ $\underline{y = -4x + 8\ln 2}$

c) at R $x + 3 = -4x + 8\ln 2$

$\therefore$ $5x = -3 + 8\ln 2$

$\therefore$ $x = -\frac{3}{5} + \frac{8}{5}\ln 2$

$= b + a\ln 2$

(10)

Hence $\underline{\underline{a = \frac{8}{5}}}$

d) $\underline{\underline{b = -\frac{3}{5}}}$

TOTAL 60.