# SECTION 1

1. $\frac{dy}{dx} = 3x^2 - 6x + 3$

stationary where $\frac{dy}{dx} = 0$ $\therefore$ $x^2 - 2x + 1 = 0$

$\therefore$ $(x-1)(x-1) = 0$

$\frac{d^2y}{dx^2} = 6x - 6$

$\therefore$ $x = 1$

at $x = 1$ $\frac{d^2y}{dx^2} = 6 \times 1 - 6 = 0$ $\therefore$ $x = 1$ is a possible point of inflection.

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2. $\frac{dy}{dx} = -\frac{1}{x^2} + 81x^2$

Stationary points at $-\frac{1}{x^2} + 81x^2 = 0$

$\therefore$ $x = \pm\frac{1}{3}$ ie $a = \frac{1}{3}$

$\frac{d^2y}{dx^2} = \frac{2}{x^3} + 162x$

$x = \frac{1}{3}$ $\frac{d^2y}{dx^2} = 2 \times 27 + 162 \times \frac{1}{3} = 108 > 0$ $\therefore$ minimum

$x = -\frac{1}{3}$ $\frac{d^2y}{dx^2} = -2 \times 27 - 162 \times \frac{1}{3} = -108 < 0$ $\therefore$ maximum

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3. midpoint of $AB = \left(\frac{-4+2}{2}, \frac{6+8}{2}\right) = (-1, 7)$

gradient of $AB = \frac{8-6}{2--4} = \frac{1}{3}$

$\therefore$ gradient of perpendicular $= -3$

$\Rightarrow \quad y - 7 = -3(x+1)$

$\therefore \quad \underline{\underline{y = -3x + 4}}$

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4. midpoint $(6, 2)$ gradient $= \frac{12}{-2} = -6$

$\therefore$ gradient of perpendicular $= \frac{1}{6}$

$y - 2 = \frac{1}{6}(x - 6)$

$\therefore \quad y = \frac{x}{6} + 1$

at Q $y = 0 \quad \therefore \; x = -6 \qquad \underline{\underline{(-6, 0)}}$

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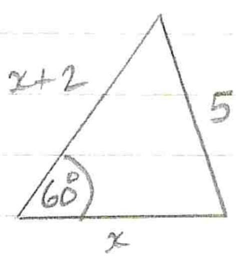
5.




Cosine rule: $5^2 = x^2 + (x+2)^2 - 2x(x+2)\cos 60$

$\therefore \; 25 = 2x^2 + 4x + 4 - 2x(x+2)\cdot\frac{1}{2}$

$\therefore \quad x^2 + 2x - 21 = 0$

$\therefore \quad x = \frac{-2 \pm \sqrt{4 + 4\times 21}}{2}$

$\therefore \quad x = -1 \pm \sqrt{22} = \underline{\underline{3.69}}$

6. $\sin^2\theta + \cos^2\theta = 1$

$\therefore \cos^2\theta = 1 - \left(\frac{3}{5}\right)^2 = \frac{16}{25}$

$\therefore \underline{\underline{\cos\theta = \frac{4}{5}}}$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{3/5}{4/5} = \underline{\underline{3/4}}$

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7. a) $\sin^2 3\theta + \cos^2 3\theta = \underline{\underline{1}}$

b) $\dfrac{\sin 2\theta}{\sqrt{1-\sin^2 2\theta}} = \dfrac{\sin 2\theta}{\cos 2\theta} = \underline{\underline{\tan 2\theta}}$

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8. $\sin\theta = 5/7 \qquad 0 < \theta \leq 360$

$\underline{\underline{\theta = 45.6^\circ, 134.4^\circ}}$

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9. $\sin(3\theta - 45) = \frac{1}{2} \qquad 0 \leq \theta \leq 180$

$\therefore 3\theta - 45 = \sin^{-1}\left(\frac{1}{2}\right) \qquad -45 \leq 3\theta - 45 \leq 495$

$\therefore 3\theta - 45 = 30, 150, 390,$

$\therefore \underline{\underline{\theta = 25^\circ, 65^\circ, 145^\circ}}$

10. $2\sin^2\theta = 3 - 3\cos\theta$ $\qquad 0 \leq \theta \leq 180$

$2(1-\cos^2\theta) = 3 - 3\cos\theta$

$\therefore 2\cos^2\theta - 3\cos\theta + 1 = 0$

let $c = \cos\theta$ $\therefore 2c^2 - 3c + 1 = 0$

$\therefore (2c-1)(c-1) = 0$

$\therefore c = \frac{1}{2}, 1$

$\therefore \cos\theta = \frac{1}{2}, 1$

$\cos\theta = \frac{1}{2}$ $\qquad \theta = 60^\circ,$

$\cos\theta = 1$ $\qquad \theta = 0^\circ$

$\underline{\underline{\theta = 0^\circ, 60^\circ}}$

# SECTION 2

1 a) $y = \dfrac{2}{\cos x} \quad \therefore \quad \dfrac{dy}{dx} = \dfrac{2\sin x}{\cos^2 x} = \underline{\underline{2\tan x \sec x}}$

b) $f(x) = \dfrac{1}{\sin 4x} \quad \therefore \quad f'(x) = -\dfrac{4\cos 4x}{\sin^2 4x} = \underline{\underline{-4\cot 4x \operatorname{cosec} 4x}}$

c) $y = \dfrac{5}{\tan \frac{\pi x}{3}} \quad \therefore \quad \dfrac{dy}{dx} = -\dfrac{\frac{5\pi}{3}\sec^2 \frac{\pi x}{3}}{\tan^2 \frac{\pi x}{3}} = \underline{\underline{-\dfrac{5\pi}{3}\operatorname{cosec}^2 \dfrac{\pi x}{3}}}$

d) $f(x) = \dfrac{3}{\cos(x-3)} \quad \therefore \quad f'(x) = \dfrac{3\sin(x-3)}{\cos^2(x-3)} = \underline{\underline{3\tan(x-3)\sec(x-3)}}$

e) $y = \dfrac{1}{\tan(2x-3)} \quad \therefore \quad \dfrac{dy}{dx} = \dfrac{-2\sec^2(2x-3)}{\tan^2(2x-3)} = \underline{\underline{-2\operatorname{cosec}^2(2x-3)}}$

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2. $y = e^x \cot x \quad \therefore \quad \dfrac{dy}{dx} = -e^x \operatorname{cosec}^2 x + e^x \cot x$

$$= e^x\left(\frac{\cos x}{\sin x} - \frac{1}{\sin^2 x}\right)$$

$$= \frac{e^x}{\sin^2 x}(\sin x \cos x - 1)$$

$\therefore \dfrac{dy}{dx} = 0$ only if $\sin x \cos x = 1$

$\therefore \sin 2x = 2$

Since we cannot have $\sin 2x > 1$ we cannot have $\dfrac{dy}{dx} = 0$ and $\therefore$ there are no turning points.

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3 a) $x = \tan^2 y \quad \therefore \quad \dfrac{dx}{dy} = 2\tan y \sec^2 y$

$\sin^2 y + \cos^2 y = 1 \quad \therefore \quad \tan^2 y + 1 = \sec^2 y$

$$\therefore \quad \frac{dx}{dy} = 2\tan y\,(1 + \tan^2 y)$$

$$= 2\sqrt{x}\,(1 + x)$$

$$\therefore \quad \frac{dy}{dx} = \frac{1}{2\sqrt{x}(1+x)}$$

b) at $y = \frac{\pi}{4} \quad x = \tan^2 \frac{\pi}{4} = 1 \quad \therefore \quad \dfrac{dy}{dx} = \frac{1}{4}$

$\therefore$ gradient of normal $= -4$

$$y - \frac{\pi}{4} = -4(x - 1)$$

$$\therefore \quad y = -4x + 4 + \frac{\pi}{4}$$

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4 a) $y = \dfrac{1}{\cos^2 2x} \quad \therefore \quad \dfrac{dy}{dx} = \dfrac{-2}{\cos^3 2x}(-2\sin 2x) = 4\tan 2x \sec^2 2x$

b) $f(x) = \dfrac{1}{\tan^3 x} \quad \therefore \quad f'(x) = \dfrac{-3}{\tan^4 x}(\sec^2 x) = -3\cot^2 x \operatorname{cosec}^2 x$

c) $y = \dfrac{1}{\sin^2(2x+1)} \quad \therefore \quad \dfrac{dy}{dx} = \dfrac{-2}{\sin^3(2x+1)}\big(2\cos(2x+1)\big) = -4\cot(2x+1)\operatorname{cosec}^2(2x+1)$

d) $f'(x) = \dfrac{4\sec^2 4x}{\tan 4x} = 4\sec^2 4x \cot 4x = 8\operatorname{cosec} 8x$

e) $\frac{dy}{dx} = 3\cos 3x\, e^{\sin 3x}$

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5. $y = \operatorname{cosec}(x - \pi/6)$ $\therefore$ $\frac{dy}{dx} = -\frac{\cos(x - \frac{\pi}{6})}{\sin^2(x - \frac{\pi}{6})}$

a) at P $x = 0$ $\therefore$ $\frac{dy}{dx} = -\frac{\cos(-\frac{\pi}{6})}{\sin^2(-\frac{\pi}{6})} = \frac{-\sqrt{3}/2}{1/4} = -2\sqrt{3}$

$y = \operatorname{cosec}(-\pi/6) = -2$

$\therefore$ gradient of normal at $P = \frac{+1}{2\sqrt{3}}$

$\therefore$ $y + 2 = \frac{+1}{2\sqrt{3}} x$

$\therefore$ $y = \frac{x}{2\sqrt{3}} - 2$

b) at Q $x = \pi/3$ $\therefore$ $y = \operatorname{cosec}(\pi/3 - \pi/6) = 2$

$\frac{dy}{dx} = -\frac{\cos(\pi/6)}{\sin^2(\pi/6)} = \frac{-\sqrt{3}/2}{1/4} = -2\sqrt{3}$

$y - 2 = -2\sqrt{3}(x - \pi/3)$

$y = -2\sqrt{3}x + 2 + \frac{2\pi\sqrt{3}}{3}$

5c) at $R$: $\frac{x}{2\sqrt{3}} - 2 = -2\sqrt{3}x + 2 + 2\pi\frac{\sqrt{3}}{3}$

$\therefore x\left(2\sqrt{3} + \frac{1}{2\sqrt{3}}\right) = 4 + 2\pi\frac{\sqrt{3}}{3}$

$\therefore x\left(\frac{(2\sqrt{3})^2+1}{2\sqrt{3}}\right) = \frac{12+2\pi\sqrt{3}}{3}$

$\therefore \frac{13x}{2\sqrt{3}} = \frac{12+2\pi\sqrt{3}}{3}$

$\therefore x = \frac{2\sqrt{3}}{13}\left(\frac{12+2\pi\sqrt{3}}{3}\right)$

$\therefore x = \frac{8\sqrt{3}+4\pi}{13}$ 11

6.a) $\frac{dx}{dt} = 3t^2$ $\frac{dy}{dt} = 1$ $\therefore \frac{dy}{dx} = \frac{1}{3t^2}$

b) $\frac{dx}{dt} = 3$ $\frac{dy}{dt} = \frac{1}{t^2}$ $\therefore \frac{dy}{dx} = \frac{1}{3t^2}$

c) $\frac{dx}{dt} = -2\sin 2t$ $\frac{dy}{dt} = \cos t$ $\therefore \frac{dy}{dx} = -\frac{\cos t}{2\sin 2t} = -\frac{1}{4}\operatorname{cosec} t$

d) $\frac{dx}{dt} = e^{t+1}$ $\frac{dy}{dt} = 2e^{2t-1}$ $\therefore \frac{dy}{dx} = 2e^{t-2}$ 7

7.a) $x = t + \frac{1}{t}$ $\therefore$ $\frac{dx}{dt} = 1 - \frac{1}{t^2}$

$y = t - \frac{1}{t}$ $\therefore$ $\frac{dy}{dt} = 1 + \frac{1}{t^2}$

$\therefore$ $\frac{dy}{dx} = \frac{t^2+1}{t^2-1}$

$t = 3$ $\therefore$ $\frac{dy}{dx} = \frac{10}{8} = \frac{5}{4}$

$x = 3 + \frac{1}{3} = \frac{10}{3}$ $\quad y = 3 - \frac{1}{3} = \frac{8}{3}$

$\therefore$ at P $\quad y - \frac{8}{3} = \frac{5}{4}\left(x - \frac{10}{3}\right)$

$\therefore$ $y = \frac{5x}{4} - \frac{3}{2}$

b) $x^2 - y^2 = \left(t + \frac{1}{t}\right)^2 - \left(t - \frac{1}{t}\right)^2$

$= \left(t^2 + \frac{1}{t^2} + 2\right) - \left(t^2 + \frac{1}{t^2} - 2\right) = 4$

$\therefore$ $x^2 - y^2 = 4$

c) $x^2 - \left(\frac{5x}{4} - \frac{3}{2}\right)^2 = 4$ $\therefore$ $x^2 - \left(\frac{25x^2}{16} + \frac{9}{4} - \frac{15x}{4}\right) = 4$

$\therefore$ $-\frac{9x^2}{16} - \frac{9}{4} + \frac{15x}{4} = 4$

$\therefore$ $9x^2 - 60x + 100 = 0$ (10)

discriminant $= (-60)^2 - 4 \times 9 \times 100 = 0$ $\therefore$ tangent touches the curve only once and does not meet the curve again