SECTION 1

1. $A = 4\pi r^2 \therefore \frac{dA}{dr} = 8\pi r$

$r = 6 \therefore \frac{dA}{dr} = 48\pi$

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2. $y = 2x^3 + x^{\frac{1}{2}} + 1 + 2x^{-1}$

$\therefore \frac{dy}{dx} = 6x^2 + \frac{1}{2}x^{-\frac{1}{2}} - 2x^{-2}$

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3. $(1-3)^2 + (-3+4)^2 = 4 + 1 = 5 = r^2 \therefore r = \sqrt{5}$

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4. $x^2 + 2x + (4x-1)^2 = k$

$\therefore 17x^2 - 6x + 1 - k = 0$

discriminant $< 0 \therefore (-6)^2 - 4 \times 17(1-k) < 0$

$\therefore 36 - 68 + 68k < 0$

$\therefore k < \frac{8}{17}$

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5. $(-2, 8)$ A
$(7, 7)$ B
$(-3, -1)$ C

gradient of AB $= \frac{8-7}{-2-7} = \frac{-1}{9}$ $\therefore$ gradient of perpendicular $= 9$
midpoint of AB $= (\frac{5}{2}, \frac{15}{2})$

5. equation of perpendicular bisector of AB is

$$y - \frac{15}{2} = 9\left(x - \frac{5}{2}\right)$$

$$\therefore \underline{y = 9x - 15.}$$

gradient of BC $= \frac{7 - -1}{7 - -3} = \frac{4}{5}$

$\therefore$ gradient of perpendicular $= -5/4$

midpoint of BC $= (2, 3)$

equation of perpendicular bisector of BC is

$$y - 3 = -\frac{5}{4}(x - 2)$$

$$\underline{y = -\frac{5x}{4} + \frac{11}{2}}$$

The 2 perpendicular bisectors meet at the centre of the circle:

$$9x - 15 = -\frac{5x}{4} + \frac{11}{2}$$

$$\therefore x = 2$$

$$\therefore y = 9 \times 2 - 15 = 3$$

ie centre of circle is $(2, 3)$

$$(x - 2)^2 + (y - 3)^2 = r^2$$

at B $(7-2)^2 + (7-3)^2 = 25 + 16 = 41 = r^2$

$$\therefore \underline{\underline{(x - 2)^2 + (y - 3)^2 = 41}}$$

6. if $(x-2)$ is a factor $f(2) = 0$

$f(2) = 2^3 + 2^2 - 4\times 2 - 4 = 8+4-8-4 = 0$

$\therefore$ $x-2$ is a factor

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7. $x-1$ is a factor $\therefore f(1) = 0$

$f(1) = 5 - 9 + 2 + a = -2 + a = 0$ $\therefore$ $a = 2$

8.

$$
\begin{array}{r l}
 & x^3 + 2x^2 - 5x + 4 \\
3x+2 \,\big) & 3x^4 + 8x^3 - 11x^2 + 2x + 8 \\
 & 3x^4 + 2x^3 \\
 & \quad 6x^3 - 11x^2 \\
 & \quad 6x^3 + 4x^2 \\
 & \qquad -15x^2 + 2x \\
 & \qquad -15x^2 - 10x \\
 & \qquad\quad 12x + 8 \\
 & \qquad\quad 12x + 8 \\
 & \qquad\qquad 0
\end{array}
$$

$x^3 + 2x^2 - 5x + 4$

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9. $a + \frac{1}{a} \geq 2$ multiply by a^2

$\therefore a^3 + a \geq 2a^2$

$\therefore a^3 - 2a^2 + a \geq 0$

9. $\therefore \quad a(a^2 - 2a + 1) \geq 0$

$\therefore \quad a(a-1)^2 \geq 0$

$(a-1)^2 \geq 0$ for all ~~but~~ a

$\therefore$ we require $a > 0$ for the inequality to be true. Note that $a \neq 0$ since we have $\frac{1}{a}$.

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10. $(p+q)^2 = p^2 + q^2 + 2pq$

$(p-q)^2 = p^2 + q^2 - 2pq$

$\therefore (p+q)^2 - (p-q)^2 = 4pq$

$\therefore \quad p+q = \sqrt{(p-q)^2 + 4pq}$

if p and q are positive then LHS > 0 (for all p and q)

$(p-q)^2 \geq 0$ for all p, q

$\therefore \quad p+q \geq \sqrt{4pq}$

and if $p \neq q$ $\quad p+q > \sqrt{4pq}$

SECTION 2

1. a) $2x + 2y\frac{dy}{dx} = 0 \quad \therefore \quad \frac{dy}{dx} = -\frac{x}{y}$

b) $2 - \frac{dy}{dx} + 2y\frac{dy}{dx} = 0 \quad \therefore \quad \frac{dy}{dx} = \frac{2}{1-2y}$

c) $\cos x - \sin y\frac{dy}{dx} = 0 \quad \therefore \quad \frac{dy}{dx} = \frac{\cos x}{\sin y}$

d) $2e^{x} - 6e^{2y}\frac{dy}{dx} = \frac{dy}{dx} \quad \therefore \quad \frac{dy}{dx} = \frac{2e^{x}}{1+6e^{2y}}$

e) $\frac{1}{x} + \frac{6}{y}\frac{dy}{dx} = 1 \quad \therefore \quad \frac{dy}{dx} = \frac{y}{6}\left(1 - \frac{1}{x}\right)$

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2. $4\sin y - \sec x = 0$

$\therefore \quad 4\cos y\frac{dy}{dx} - \frac{\sin x}{\cos^2 x} = 0$

$x = \frac{\pi}{3} \quad y = \frac{\pi}{6} \quad \therefore \quad 4\cos\frac{\pi}{6}\frac{dy}{dx} - \frac{\sin\frac{\pi}{3}}{\cos^2\frac{\pi}{3}} = 0$

$\therefore \quad \frac{4\sqrt{3}}{2}\frac{dy}{dx} - \frac{\sqrt{3}/2}{1/4} = 0$

$\therefore \quad \frac{dy}{dx} = 1$

$\therefore \quad y - \frac{\pi}{6} = x - \frac{\pi}{3}$

$\therefore \quad y = x - \frac{\pi}{6}$

(5)

3a) $3x^2y + x^3\frac{dy}{dx} = 0 \quad \therefore \quad \frac{dy}{dx} = -\frac{3y}{x}$

b) $e^{2y} + 2xe^{2y}\frac{dy}{dx} - \frac{1}{x} - \frac{1}{y}\frac{dy}{dx} = 0$

$\therefore \quad \frac{dy}{dx}\left(2xe^{2y} - \frac{1}{y}\right) = \frac{1}{x} - e^{2y}$

$\therefore \quad \frac{dy}{dx}\left(\frac{2xye^{2y} - 1}{y}\right) = \frac{1 - xe^{2y}}{x}$

$\therefore \quad \frac{dy}{dx} = \frac{y(1 - xe^{2y})}{x(2xye^{2y} - 1)}$

c) $\sin y + x\cos y\frac{dy}{dx} + 2y\frac{dy}{dx}\operatorname{cosec} x - \frac{y^2\cos x}{\sin^2 x} = 0$

$\therefore \quad \frac{dy}{dx}\left(x\cos y + 2y\operatorname{cosec} x\right) - y^2\cot x\operatorname{cosec} x + \sin y = 0$

$\therefore \quad \frac{dy}{dx} = \frac{y^2\cot x\operatorname{cosec} x - \sin y}{x\cos y + 2y\operatorname{cosec} x}$

d) $x\frac{dy}{dx} + y - \cos x = e^y\frac{dy}{dx}$

$\therefore \quad \frac{dy}{dx}\left(e^y - x\right) = y - \cos x$

$\therefore \quad \frac{dy}{dx} = \frac{y - \cos x}{e^y - x}$

3e) $$\frac{1}{x+2} = \frac{2}{2y+1}\frac{dy}{dx}$$

$$\therefore \frac{dy}{dx} = \frac{1}{2}\left(\frac{2y+1}{x+2}\right)$$

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4. $$3^x + y^2 = (x+3)y$$

$$\therefore 3^x \ln 3 + 2y\frac{dy}{dx} = (x+3)\frac{dy}{dx} + y$$

$$\therefore \frac{dy}{dx}\left(x+3-2y\right) = 3^x \ln 3 - y$$

$$\therefore \frac{dy}{dx} = \frac{3^x \ln 3 - y}{x+3-2y}$$

at $(1, 1)$ $$\frac{dy}{dx} = \frac{3\ln 3 - 1}{1+3-2} = \tfrac{1}{2}(3\ln 3 - 1)$$

$\therefore$ gradient of normal $= \dfrac{2}{1-3\ln 3}$

$$y - 1 = \frac{2}{1-3\ln 3}(x-1)$$

(5)

5a) $\frac{dy}{dx} = 4x^3 - 108x$ $\qquad \frac{d^2y}{dx^2} = 12x^2 - 108$

point of inflection where $\frac{d^2y}{dx^2} = 0$ $\quad \therefore 12x^2 - 108 = 0$

$\therefore x = \pm 3.$

$x < -3$ $\quad \frac{d^2y}{dx^2} > 0$

$x > -3$ and $x < 3$ $\quad \frac{d^2y}{dx^2} < 0$

$x > 3$ $\quad \frac{d^2y}{dx^2} > 0$

Hence, $x < -3$ or $x > 3 \Rightarrow y$ is convex

$-3 < x < 3 \Rightarrow y$ is concave.

b) $\frac{dy}{dx} = e^{-x} - xe^{-x}$ $\qquad \frac{d^2y}{dx^2} = -e^{-x} - e^{-x} + xe^{-x}$

$= -2e^{-x} + xe^{-x}$

for $\frac{d^2y}{dx^2} = 0$ $\quad xe^{-x} = 2e^{-x}$

$\therefore x = 2$ (since $e^{-x} \neq 0$)

$x < 2$ $\quad \frac{d^2y}{dx^2} < 0$ $\qquad x < 2$ y is concave

$x > 2$ y is convex.

$x > 2$ $\quad \frac{d^2y}{dx^2} > 0$

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6. $y = e^{x} \sin x$ $\quad\quad$ $\frac{dy}{dx} = e^{x} \cos x + e^{x} \sin x$ ✓

$$\frac{d^2y}{dx^2} = e^{x} \cos x - e^{x} \sin x + e^{x} \sin x + e^{x} \cos x$$

$$= 2e^{x} \cos x$$ ✓

$$\therefore \frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 2e^{x} \cos x - 2\left(e^{x} \cos x + e^{x} \sin x\right) + 2e^{x} \sin x$$ ✓

$$= \underline{0}$$ QED.

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7. $\frac{dh}{dt} = 0.6$ ✓ $\quad\quad$ $\frac{dV}{dh} = 0.2 \times 10\pi e^{0.2h}$

$$= 2\pi e^{0.2h}$$ ✓

$$\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt} = 2\pi e^{0.2h} \times 0.6$$ ✓

$h = 5$ $\quad \therefore \frac{dV}{dt} = 1.2\pi e^{0.2 \times 5} = 1.2\pi e$ ✓

$$\simeq \underline{\underline{10.3 \text{ cm s}^{-1}}}$$

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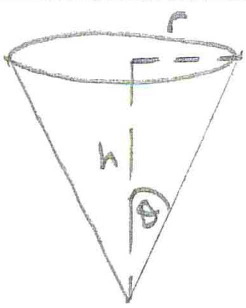
8. 

$V = \frac{1}{3}\pi r^2 h$

$\tan\theta = \frac{r}{h}$

$\therefore V = \frac{1}{3}\pi h^3 \tan^2\theta$ ✓

$\therefore \frac{dV}{dh} = \pi h^2 \tan^2\theta$ ✓

$$\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$$

$$\therefore 10 = \pi h^2 \tan^2\theta \times \frac{dh}{dt}$$

$$\therefore \frac{dh}{dt} = \frac{10}{\pi h^2 \tan^2\theta}$$

When $t=5$ $h=20$ and $V = 5\times10 = 50$

$\therefore$ for $V = \frac{1}{3}\pi h^3 \tan^2\theta \Rightarrow 50 = \frac{1}{3}\pi \times 20^3 \tan^2\theta$

$$\therefore \tan^2\theta = \frac{150}{\pi\times 20^3}$$

hence $\frac{dh}{dt} = \frac{10\pi\times 20^3}{\pi\times150\,h^2} = \frac{1600}{3h^2}$

$\therefore$ for $h=10$ $\frac{dh}{dt} = \frac{1600}{3\times10^2} = \frac{16}{3} \approx \underline{5.33\ \text{cms}^{-1}}$

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9. $N = 500\times 1.05^{0.4t}$

$$\therefore \frac{dN}{dt} = 0.4\times500\times1.05^{0.4t} \times \ln 1.05$$
$$= 0.4\times\ln 1.05\times N$$

$N=2000 \therefore \frac{dN}{dt} = 0.4\times\ln1.05\times2000$

$\simeq \underline{39\ \text{min}^{-1}}$

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TOTAL 63