PURE 20 SECTION 1

1. $\int 2x^{-3} - 3x^{\frac{1}{2}} \, dx = \underline{\underline{-x^{-2} - 2x^{3/2}}} \;(+c)$

2. $\int 4x^2 + x^{-3/2} + 5x^{-2} \, dx = \underline{\underline{\frac{4x^3}{3} - 2x^{-1/2} - 5x^{-1}}} \;(+c)$

3. $f(x) = \int f'(x) \, dx = \int 9x^2 + 4x - 3 \, dx$

$= 3x^3 + 2x^2 - 3x + c$

$f(-1) = 0 \quad \therefore \quad -3 + 2 + 3 + c = 0$

$\therefore \quad c = -2$

$\underline{\underline{f(x) = 3x^3 + 2x^2 - 3x - 2}}$

4. $\int_1^3 x^2 + 2x \, dx = \left[\frac{x^3}{3} + x^2\right]_1^3 = \underline{\underline{\frac{50}{3}}}$

5. $\int_1^4 2x^{-2} + x^{-3/2} \, dx = \left[-2x^{-1} - 2x^{-1/2}\right]_1^4 = \underline{\underline{\frac{5}{2}}}$

6. $\int_1^k x^{-1/2} \, dx = \left[2x^{\frac{1}{2}}\right]_1^k = 2\sqrt{k} - 2 = 3$

$\therefore \quad \underline{\underline{k = \frac{25}{4}}}$

7. Area $= \int_{-4}^{1} 4 - 3x - x^2 \, dx = \left[4x - \frac{3x^2}{2} - \frac{x^3}{3} \right]_{-4}^{1}$

$= \underline{\underline{\frac{125}{6}}}$

8. $y = x(x^2 - 5x + 6) = x(x-2)(x-3)$

$\therefore$ roots at $\underline{\underline{x = 0, 2, 3}}$

Total area = area from $x = 0$ to $x = 2$ plus
area from $x = 2$ to $x = 3$

Area from $x = 0$ to $x = 2$ $= \int_0^2 x^3 - 5x^2 + 6x \, dx = \left[\frac{x^4}{4} - \frac{5x^3}{3} + 3x^2 \right]_0^2 = 8/3$

Area from $x = 2$ to $x = 3$ $= \left[\frac{x^4}{4} - \frac{5x^3}{3} + 3x^2 \right]_2^3 = -5/12$

$\therefore$ Total area $= 8/3 + 5/12 = \underline{\underline{\frac{37}{12}}}$

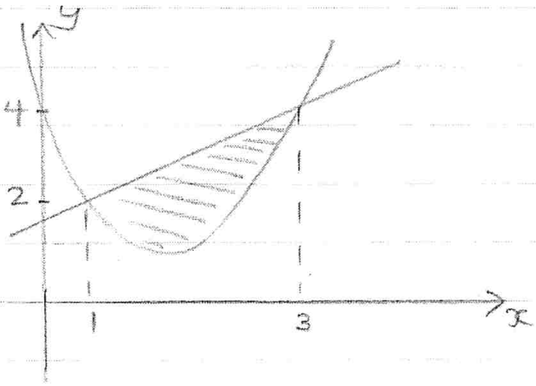
9. at points of intersection
$x^2 - 3x + 4 = x + 1$

$\therefore \; x^2 - 4x + 3 = 0$

$\therefore \; (x-3)(x-1) = 0$

$\therefore \; x = 3, 1$

$\therefore$ points of intersection are $\underline{(1, 2)\ ,\ (3, 4)}$



Area of trapezium $= \frac{2}{2}(2+4) = 6$

Area under curve $= \int_1^3 x^2 - 3x + 4 \, dx = \left[\frac{x^3}{3} - \frac{3x^2}{2} + 4x\right]_1^3$

$= \frac{14}{3}$

$\therefore$ Required area $= 6 - \frac{14}{3} = \underline{\frac{4}{3}}$

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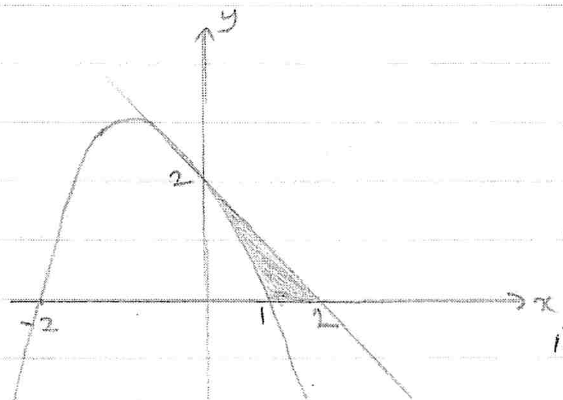
10. $y = 2 - x - x^2$ $\therefore$ at $y = 0$ $\quad x^2 + x - 2 = 0$

$\therefore (x+2)(x-1) = 0$

$\therefore x = -2, 1$ (roots)



$\frac{dy}{dx} = -1 - 2x$

on y-axis $x = 0$ $\therefore \frac{dy}{dx} = -1$

ie tangent has gradient $= -1$ and passes through $(0, 2)$

Area under curve $= \int_0^1 2 - x - x^2 \, dx$

$= \left[2x - \frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 = \frac{7}{6}$

$\therefore$ equation of tangent is $y - 2 = -x$

$\therefore y = 2 - x$

$\therefore x$ intercept is $(2, 0)$

$\therefore$ Area of triangle $= \frac{1}{2} \times 2 \times 2 = 2$.

$\therefore$ Shaded area $= 2 - \frac{7}{6} = \underline{\frac{5}{6}}$

Pure 20 – Section 2

1. (a) e^x (b) $\ln|x|$ (c) $\sin x$ (d) $\tan x$ (e) $-\text{cosec}\, x$

2. (a) $5x - 3\ln x$ (b) $\frac{2e^x}{5} + \frac{x}{5}$ (c) $3x + \ln x$

   (d) $\int \frac{\sin x}{\cos^2 x}.dx = \int \sec x \tan x \,.dx = \sec x$

   (e) $\int \frac{\cos x}{\sin^2 x}.dx = \int \text{cosec}\, x \cot x\,.dx = -\text{cosec}\, x$

3. (a) $\int (x-2)^7.dx = \frac{1}{8}(x-2)^8$

   (b) $\int (2x+5)^3.dx = \frac{1}{4\times 2}(2x+5)^4 = \frac{1}{8}(2x+5)^4$

   (c) $\int (4x-1)^{\frac{1}{2}}.dx = \frac{1}{\frac{3}{2}\times 4}(4x-1)^{\frac{3}{2}} = \frac{1}{6}(4x-1)^{\frac{3}{2}}$

   (d) $\int (\frac{x}{4}-2)^5.dx = \frac{1}{6\times\frac{1}{4}}(\frac{x}{4}-2)^6 = \frac{2}{3}(\frac{x}{4}-2)^6$

   (e) $\int 5(3-2x)^{-2}.dx = \frac{1}{-1\times -2}\times 5(3-2x)^{-1} = \frac{5}{2}(3-2x)^{-1}$

4. (a) $\frac{1}{2}\int \frac{2}{2x-1}.dx = \frac{1}{2}\ln|2x-1|$

   (b) $\frac{2}{3}\int \frac{3}{3x+5}.dx = \frac{2}{3}\ln|3x+5|$

   (c) $-\frac{3}{7}\int \frac{-7}{2-7x}.dx = -\frac{3}{7}\ln|2-7x|$

5. (a) $\frac{1}{5}\sin(5x-2) + 2e^{x+3}$

   (b) $-\frac{1}{2}e^{5-2x} + \frac{1}{3}\tan 3x$

   (c) $(e^{2x}+1)^2 = e^{4x} + 1 + 2e^{2x}$

   Hence $-\frac{5}{2}\cos(2x+3) + \frac{3}{5}\ln|5x-1| + \frac{1}{4}e^{4x} + x + e^{2x}$

6. (a) 0.75

   (b) $0.75 \times (4.25^2 - 4\times 4.25 + 5) = 4.55$ (2 d.p.)

   (c) the limit of the sum of the areas of rectangles as the width tends to zero..so the exact area under the curve

   (d) $\int_2^5 x^2 - 4x + 5.dx = [\frac{x^3}{3} - 2x^2 + 5x]_2^5 = 12$

TOTAL 52 MARKS