Parametric Integration Solutions

Section 1

① $dx/dt = (2t-1)^2 \Rightarrow x = \frac{1}{6}(2t-1)^3 + c$

$$0 = \frac{1}{6}(5)^3 + c \Rightarrow c = \frac{-125}{6}$$

At $t = 6$, $x = \frac{1}{6}(11)^3 - \frac{125}{6} = \underline{\underline{201}}$

② a) $f(t) = \int (6+5t)dt = 6t + \frac{5}{2}t^2 + c$

$0 = 0 + c \Rightarrow c = 0 \qquad \Rightarrow f(t) = 6t + \frac{5}{2}t^2$

b) $200 = 6t + \frac{5}{2}t^2 \qquad \Rightarrow 5t^2 + 12t - 400 = 0$

$\Rightarrow t = \underline{7.82} \text{ sec} \quad (\text{or } -10.22)$

Section 2

① a) $x = 0 \Rightarrow 2t - 4 = 0 \qquad \Rightarrow t = 2$ ✓

$x = 2 \Rightarrow 2t - 4 = 2 \qquad \Rightarrow t = 3$ ✓

b) Area $= \int_2^3 y\frac{dx}{dt} dt = \int_2^3 \frac{1}{t}(2)dt = \int_2^3 \frac{2}{t} dt$

c) $\left[2\ln|t|\right]_2^3 = 2\ln 3 - 2\ln 2 = 2\ln 3/2 = \ln^9/4$

② a.) A : $x = 0 \Rightarrow 4\cos\theta = 0 \Rightarrow \theta = \pi/2$ or $3\pi/2$

$\qquad y = 2\sin \pi/2 = 2 \qquad \Rightarrow$ A is when $\underline{\theta = \pi/2}$

$\qquad$ or $y = 2\sin 3\pi/2 = -2$

B : $y = 0 \Rightarrow 2\sin\theta = 0 \Rightarrow \theta = 0$ or π

$\qquad x = 4\cos 0 = 4 \qquad \Rightarrow$ B is when $\underline{\theta = 0}$

$\qquad$ or $x = 4\cos\pi = -4$

b.) Area $= \displaystyle\int_0^4 y \, dx = \int_{\frac{\pi}{2}}^0 y \, \frac{dx}{dt} \, d\theta$ ✓

$\qquad = \displaystyle\int_{\frac{\pi}{2}}^0 (2\sin\theta)(-4\sin\theta) \, d\theta$ ✓

$\qquad = \displaystyle\int_{\frac{\pi}{2}}^0 -8\sin^2\theta \, d\theta = \int_0^{\pi/2} 8\sin^2\theta \, d\theta$ ✓

c.) Area of ellipse $= \displaystyle 4\int_0^{\pi/2} 8\sin^2\theta \, d\theta = 4\int_0^{\pi/2} 8\left(\frac{1-\cos 2\theta}{2}\right) d\theta$ ✓

$\qquad = \displaystyle 4\int_0^{\pi/2} (4 - 4\cos 2\theta) \, d\theta$ ✓

$\qquad = \displaystyle 4\left[4\theta - 2\sin 2\theta\right]_0^{\pi/2}$ ✓

$\qquad = 4\left(2\pi - 0\right) - 0 \qquad = 8\pi$ ✓

③ a.) $y = 5a - x$ $\Rightarrow$ $4x/t = 5a - xt$ ✓ $\Rightarrow$ $4/t = 5 - t$

$\Rightarrow$ $4 = 5t - t^2$ $\Rightarrow$ $t^2 - 5t + 4 = 0$ ✓

$(t-1)(t-4) = 0$ $\Rightarrow$ $\underline{t = 1 \text{ or } t = 4}$

✓

$t = 1 : x = a, y = 4a$

$t = 4 : x = 4a, y = a$ so   S is $(a, 4a)$ T is $(4a, a)$

✓   ✓

b.) $\int y \frac{dx}{dt} dt = \int \frac{4a}{t}(a) \, dt = \int \frac{4a^2}{t} dt = 4a^2 \ln t + c$

✓   ✓

c.) Area between curve and line = area below line − area below curve ✓

$= \frac{1}{2}(a + 4a)(3a) - \int_1^4 \frac{4a^2}{t} dt$

✓   ✓

$= \frac{15a^2}{2} - \left[4a^2 \ln t \right]_1^4$

$= \frac{15a^2}{2} - 4a^2 \ln 4$

✓   ✓

$= \frac{a^2}{2}(15 - 8 \ln 4)$