Statistics 11 - Discrete Random Variables and Binomial Distribution

Please **complete** this homework by _____. Start it early. If you can't do a question you will then have time to ask your teacher for help or go to a drop in session.

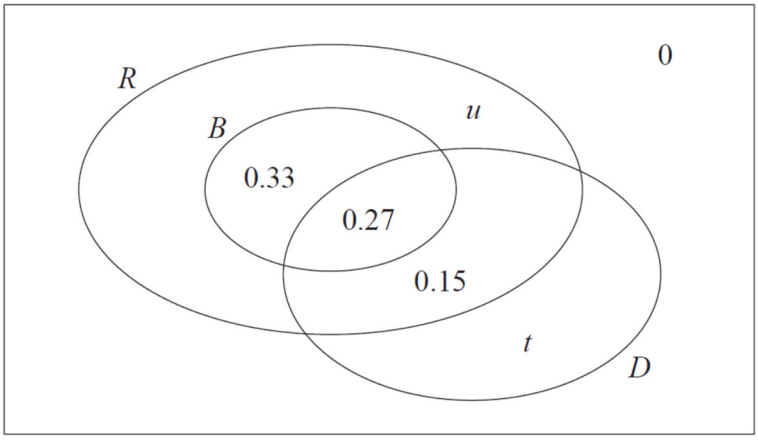
Section 1 – Review of previous topics. Please complete all questions.

1. The basic weekly earnings, to the nearest £, of 150 workers in a factory are shown in the following table:

Weekly earnings (£)	100-109	110-129	130-149	150-174	175-204
Number of plots	12	35	62	28	13

a) Calculate estimates of the mean and standard deviation of this distribution
b) Following pay negotiations workers are to receive a 3% increase in basic pay plus a fixed lump sum increase of £156. Find the mean and standard deviation of the increased weekly earnings

2. The Venn diagram shows the probabilities of customer bookings at Harry's hotel.
R is the event that a customer books a room
B is the event that a customer books breakfast
D is the event that a customer books dinner
u and *t* are probabilities.



a) Write down the probability that a customer books breakfast but does not book a room.
b) Given that the events *B* and *D* are independent, find the value of *t*.
c) Hence find the value of *u*.

Section 2 – Consolidation of this week's topic. Please complete all questions.

1. The discrete random variable *X* can take only the values 2, 3, 4 or 6. For these values the probability distribution function is given by:

x	2	3	4	6
$P(X=x)$	$\frac{5}{21}$	$\frac{2k}{21}$	$\frac{7}{21}$	$\frac{k}{21}$

where *k* is a positive integer

a) Show that $k=3$ (1)

b) Find $P(X\leq 3)$ (2)

2. A discrete random variable *X* has the probability function

$$P(X=x) \begin{cases} k(1-x)^2 & x=-1,0,1,2 \\ 0 & \text{otherwise} \end{cases}$$

Find the probability distribution for X (3)

3. The discrete random variable *X* has the probability function

$$P(X=x)=\begin{cases} kx & x=2,4,6 \\ k(x-2) & x=8 \\ 0 & \text{otherwise} \end{cases}$$

where *k* is a constant.

Find the exact value of $P(X\leq 5)$. (4)

4. The discrete random variable *X* has probability distribution

x	-4	-2	1	3	5
$P(X=x)$	0.4	p	0.05	0.15	p

a) Show that $p=0.2$ (2)

b) Find $P(X\geq 0)$ (2)

c) Find $P\left(1<\frac{10x-1}{20}<3\right)$ (3)

5. 2 fair coins are flipped. Mark is interested in X, where X = the number of tails obtained. Is X an example of the uniform distribution? Give reasons. (2)

6. The random variable $X \sim B\left(8,\frac{1}{3}\right)$. Use your calculator to find:

a) $P(X=2)$ (1)

b) $P(X\leq 2)$ (2)

c) $P(X=5)$ (1)

7. In a particular profession, 60 per cent of candidates pass a test. A group of 12 people sit the test. Find the probability that

a) Half of the group pass the test (2)

b) Three quarters of the group pass the test (1)

c) Between half and three quarters (inclusive) pass the test (3)

8. On a day with light rain forecast, one fifth of people have an umbrella with them. Of 16 people waiting at a bus stop find the probability that:

a) Four people have an umbrella (2)
b) Ten people have an umbrella (1)
c) Seven people do not have an umbrella (1)

9. A football team lose 50% of games, and draw 15% of games. Assuming the binomial model is appropriate, find the probability that over a 10 game tournament they:

a) Win no more than two games (3)
b) Win half of the games (1)
c) The team plays three 10 game tournaments over the season. Find the probability they win no more than two games in exactly one of these tournaments (2)
d) Explain why the use of the binomial model is suspect for all parts of this question (1)

(Total 40 Marks)

Section 3 – Extension question. If you are aiming for a top grade, you should attempt these questions.

1. The discrete random variable X has the probability distribution

x	1	2	3	4
$P(X=x)$	k	$2k$	$3k$	$4k$

a) Show that $k = 0.1$
b) Find $P(X< 3)$
c) Two independent observations X_1 and X_2 are made of X. Show that $P(X_1 + X_2 = 4) = 0.1$
d) Complete the probability distribution table for $X_1 + X_2$

y	2	3	4	5	6	7	8
$P(X_1 + X_2 = y)$	0.01	0.04	0.10		0.25	0.24	

e) Find $P(1.5 < X_1 + X_2 \leq 3.5)$

2. State the conditions under which the binomial distribution may be used. Records kept in a hospital show that 3 out of every 10 casualties who come to the casualty department have to wait more than half an hour. Find the most probable number of the first 8 casualties that will have to wait more than half an hour.