Godalming College

Statistics 22 Normal distribution (approximating binomial)

Please **complete** this homework by _____. Start it early. If you can't do a question you will then have time to ask your teacher for help or go to a drop-in session.

Section 1 – Review of previous topics. Please complete all questions.

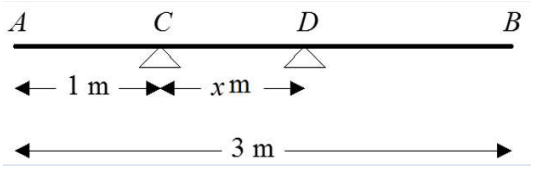
1. The continuous random variable Y is normally distributed with mean 40 and variance 9.

   Find k such that $P(40 - k < Y < 40 + k) = 0.668$

2. Data is collcted on the temperture in $°C$ of a chemical reaction (t) and the amount of dry residue produced (d grams)
   The data are coded using the changes of variable $x = t$ and $y = logd$. The regression line of y on x is fund to be $y = -0.785 + 0.123x$.
   Given that the data can be modelled by an equation of the form $d = ab^t$, where a and b are constatnts, find the values of a and b.

3. In a factory, machines A, B and C are all producing metal rods of the same length. Machine A produces 35% of the rods, machine B produces 25% and the rest are produced by machine C. Of their production of rods, machines A, B and C produce 3%, 6% and 5% defective rods respectively.
   a) Draw a tree diagram to represent this information
   b) Find the probability that a randomly selected rod is
      i) produced by machine A and is defective,
      ii) is defective
   c) Given that a randomly selected rod is defective, find the probability that it was produced by machine C.

4. A uniform plank AB has weight 120N and length 3m. The plank rests horizontally in equilibrium on two smooth supports C and D, where AC = 1m and CD = x m, as shown in the diagram. The reaction of the support on the plank at D has magnitude 80N.



   Modelling the plank as a rod,
   a) Show that $x = 0.75$

   A rock is now placed at B, and the plank is on the point of tilting about D. Modelling the rock as a particle, find:

   b) The weight of the rock,
   c) The magnitude of the reaction of the support on the plank at D
   d) State how you have used the model of the rock as a particle.

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Section 2 – Consolidation of this week's topic. Please complete all questions.

1. For each of the following binomial random variables, X;
   i) State, with reasons, whether X can be approximated by a normal distribution.
   ii) If appropriate, write down the normal approximation to X in the form $N(\mu, \sigma^2)$, giving the values of μ and σ.
   a) $X \sim B(150, 0.6)$ b) $X \sim B(30, 0.4)$
   c) $X \sim B(250, 0.8)$ d) $X \sim B(800, 0.47)$ *(8 marks)*

2. The random variable $Y \sim B(120, 0.54)$. Use a suitable approximation to estimate:
   a) $P(Y \leq 55)$ *(3 marks)*
   b) $P(Y > 70)$ *(2 marks)*
   c) $P(60 < Y \leq 75)$ *(3 marks)*

3. The random variable $X \sim B(250, 0.4)$. Use a suitable approximation to estimate:
   a) $P(90 < X \leq 100)$ *(3 marks)*
   b) $P(X = 130)$ *(3 marks)*

4. In a large college 58% of students are female and 42% are male. A random sample of 100 students is chosen from the college.
   a) Using a suitable approximation find the probability that more than half the sample are female.
   b) Find the percentage error when using a normal approximation to calculate the probability that more than half the sample are female. *(9 marks)*

5. The discrete random variable X is distributed $B(n, p)$
   a) Write down the value of p that will give the most accurate estimate when approximating the binomial distribution by a normal distribution. *(1 mark)*
   b) Give a reason to support your value. *(1 mark)*
   c) Given that n=200 and p=0.48, find $P(90 \leq X < 105)$ *(5 marks)*

6. The sale staff at an insurance company make house calls to prospective clients. Past records show that 30% of the people visited will take out a new policy with the company.
   On a particular day, one salesperson visits 8 people. Find the probability that, of these,
   a) Exactly 2 take out new policies, *(2 marks)*
   b) More than 4 take out new policies *(2 marks)*

   The company awards a bonus to any salesperson who sells more than 50 policies in a month.

   c) Using a suitable approximation, find the probability that a salesperson gets a bonus in a month in which he visits 150 prospective clients. *(4 marks)*

7. For a particular type of plant 45% have white flowers and the remainder have coloured flowers. Gardenmania sells plants in batches of 12. A batch is selected at random. Calculate the probability that this batch contains
   a) Exactly 5 plants with white flowers *(2 marks)*
   b) More plants with white flowers than coloured ones *(2 marks)*

   Gardenmania takes a random sample of 10 batches of plants.

   c) Find the probability that exactly 3 of these batches contain more plants with white flowers than coloured ones. *(2 marks)*

   Due to an increasing demand for these plants by large companies, Gardenmania decides to sell them in batches of 50.

   d) Use a suitable approximation to calculate the probability that a batch of 50 plants contains more than 25 plants with white flowers. *(4 marks)*

(Total = 56 marks)