

Statistics 23 – Normal distribution (Hypothesis testing)

Please **complete** this homework by _____. Start it early. If you can't do a question you will then have time to ask your teacher for help or go to a drop-in session.

Section 1 – Review of previous topics. Please complete all questions.

1. The random variable B is normally distributed with a standard deviation of 7.2. Given also that $P(B > 110) = 0.138$, find the mean of B .

2. A beam AB has length 6m and weight 200N. The beam rests in a horizontal position on two supports at the points C and D , where $AC = 1\text{m}$ and $DB = 1\text{m}$. Two children, Sophie and Tom, each of weight 500N, stand on the beam with Sophie standing twice as far from the end B as Tom. The beam remains horizontal and in equilibrium and the magnitude of the reaction at D is three times the magnitude of the reaction at C . By modelling the beam as a uniform rod and the two children as particles, find how far Tom is standing from the end B .

3. For the events A and B ,

$$P(A \cap B') = 0.32, P(A' \cap B) = 0.11 \text{ and } P(A \cup B) = 0.65$$

- Draw a Venn diagram to illustrate the complete sample space for the events A and B .
 - Write down the value of $P(A)$ and the value of $P(B)$
 - Find $P(A/B')$
 - Determine whether or not A and B are independent.
4. A geologist is analysing the size of quartz crystals in a sample of granite. She estimates that the longest diameter of 75% of the crystals is greater than 2mm, but only 10% of the crystals have a longest diameter of more than 6mm.

The geologist believes that the distribution of the longest diameters of the quartz crystals can be modelled by a normal distribution.

- Find the mean and variance of this normal distribution.

The geologist also observed from her sample that only 2% of the longest diameters were smaller than 1mm

- Calculate the corresponding percentage that would be predicted by a normal distribution with the parameters calculated in part a)
- Hence, comment on the suitability of the normal distribution as a model in this situation.

Section 2 – Consolidation of this week’s topic. Please complete all questions

1. A report states that employees spend, on average, 80 minutes every working day on personal use of the internet. A company takes a random sample of 100 employees and finds their mean personal internet use (in a day) is 83 minutes with a standard deviation of 15 minutes. The company’s managing director claims that his employees spend more time on average on personal use of the internet than the report states.

Test, at the 5% level of significance, the managing directors’s claim. State your hypothesis clearly. (7 marks)

2. The weight of a plastic box manufactured by a company is W grams, where $W \sim N(\mu, 20.25)$. A significance test of the null hypothesis $H_0: \mu = 50.0$, against the alternative hypothesis $H_1: \mu \neq 50.0$, is carried out at the 5% significance level, based on a sample of size n .

Given that $n=81$, find the critical region for the test, in terms of the sample mean \bar{W} (5 marks)

3. The random variable U has the distribution $N(\mu, \sigma^2)$, where the value of σ is known. A test is carried out of the null hypothesis $H_0: \mu = 50.0$ against the alternative hypothesis $H_1: \mu > 50.0$. The tests is carried out at the 1% significance level and is based on a random sample of size 10.

The test is carried out once. The value of the sample mean is 53.0. The outcome of the test is that H_0 is not rejected. Show that $\sigma > 4.08$, correct to 3 significant figures. (4 marks)

4. Over a long period the number of vistors per week to a stately home was known to have the distribution $N(500, 100^2)$. After higher car parking charges were introduced, a sample of four randomly chosen weeks gave a mean number of visitors per week of 435. You should assume that the number of visitors per week is still normally distributed with variance 100^2 .

Test, at the 10% significance level, whether there is evidence that the mean number of vistors per week has fallen. (7 marks)

(Total 23 marks)