Statistics 11 - Discrete Random Variables & Binomial Distribution - Solutions

Section 1.

1a) Midpoints : 104.5, 119.5, 139.5, 162, 189.5

From calc mean = £140.57

s.dev = £22.39

b) New mean = $1.03 \times 140.57 + 156 =$ £300.78

New s.dev = $1.03 \times 22.39 =$ £23.06

2 a) 0

b) $P(B \cap D) = P(B) \times P(D)$

$0.27 = (0.33 + 0.27) \times (0.27 + 0.15 + t)$

$\Rightarrow 0.27 = 0.6(0.42 + t)$

$\Rightarrow t = 0.03$

c) $u = 1 - 0.33 - 0.27 - 0.15 - 0.03 = 0.22$

Section 2.

1.a) $\frac{5}{21} + \frac{2k}{21} + \frac{7}{21} + \frac{k}{21} = 1$

$\Rightarrow 5 + 2k + 7 + k = 21$

$\Rightarrow 3k + 12 = 21$

$\Rightarrow 3k = 9$

$\Rightarrow k = 3$

b) $P(X \le 3) = P(X=2) + P(X=3) = \frac{5}{21} + \frac{2k}{21} = \frac{5}{21} + \frac{6}{21} = \frac{11}{21}$ (3)

2.

X	-1	0	1	2
$P(X)$	$4k$	k	0	k

$\Rightarrow 6k = 1 \Rightarrow k = \frac{1}{6}$

$\Rightarrow$ distribution is

X	-1	0	1	2
$P(X)$	$\frac{2}{3}$	$\frac{1}{6}$	0	$\frac{1}{6}$

(3)

3.

X	2	4	6	8
$P(X)$	$2k$	$4k$	$6k$	$6k$

$\Rightarrow 18k = 1 \Rightarrow k = \frac{1}{18}$

$P(X \le 5) = P(X=2) + P(X=4)$

$= 2k + 4k = 6k = 6\left(\frac{1}{18}\right) = \frac{1}{3}$ (4)

4. a) $0.4 + p + 0.05 + 0.15 + p = 1$

$\Rightarrow 2p + 0.6 = 1$

$\Rightarrow 2p = 0.4$

$\Rightarrow p = 0.2$

b) $P(X \ge 0) = P(X=1) + P(X=3) + P(X=5)$

$= 0.05 + 0.15 + p = 0.2 + 0.2 = 0.4$

c) $1 < \frac{10x-1}{20} < 3$

$\Rightarrow 20 < 10x - 1 < 60$

$\Rightarrow 21 < 10x < 61$

$\Rightarrow 2.1 < x < 6.1$

$P\left(1 < \frac{10x-1}{20} < 3\right) = P(2.1 < x < 6.1) = P(X=3) + P(X=5)$

$= 0.15 + p = 0.15 + 0.2$

$= 0.35$ (7)

5. 4 options HH, HT, TH, TT

$P(X=0) = P(HH) = \frac{1}{4}$

$P(X=1) = P(HT) + P(TH) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

$\Big\}$ probabilities are different $\Rightarrow X$ is not uniform. (2)

6. a) $P(X=2) = 0.2731$

b) $P(X \le 2) = P(X=0) + P(X=1) + P(X=2) =$

$= 0.0390 + 0.1561 + 0.2731 = 0.4682$

c) $P(X=5) = 0.0683$ (4)

7. X = no. that pass the test

$X \sim Bin(12, 0.6)$

a) $P(X = 6) = 0.1766$

b) $P(X = 9) = 0.1419$

c) $P(6 \le X \le 9) = P(X=6) + P(X=7) + P(X=8) + P(X=9)$

$= 0.1766 + 0.2270 + 0.2128 + 0.1419 = 0.7583$

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8. U = no. with Umbrella

$U \sim Bin(16, \frac{1}{5})$

a) $P(U=4) = 0.2001$

b) $P(U=10) = 0.0002$

c) $P(U=9) = 0.001228$

16−7=9

or D = no. don't have Umbrella

$D \sim Bin(16, \frac{4}{5})$

$P(D=7) = 0.001228$

4

9. W = no. games won

100% − 50% − 15% = 35%

$W \sim Bin(10, 0.35)$

a) $P(W \le 2) = P(W=0) + P(W=1) + P(W=2)$

$= 0.0135 + 0.0725 + 0.1757 = 0.2616$

b) $P(W=5) = 0.1536$

c) T = no. tournaments they win no more than 2 games in

$T \sim Bin(3, 0.2616)$

$P(T=1) = 0.4279$

d) Winning games will depend on opposition i.e not independent, p not fixed, binomial should not be used.

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TOTAL = 40

Section 3.

1\. a) $k + 2k + 3k + 4k = 1 \Rightarrow 10k = 1 \Rightarrow k = 0.1$

b) $P(X<3) = P(X=1) + P(X=2) = k + 2k = 3k = 3(0.1) = 0.3$

c) $P(X_1 + X_2 = 4) = P(X_1=1 \cap X_2=3) + P(X_1=2 \cap X_2=2) + P(X_1=3 \cap X_2=1)$

$= k \times 3k + 2k \times 2k + 3k \times k$

$= 3k^2 + 4k^2 + 3k^2$

$= 10k^2 = 10(0.1)^2 = 0.1$

d) $P(X_1 + X_2 = 5) =$

X_1	X_2		
1	4	$k \times 4k$	$= 4k^2$
2	3	$2k \times 3k$	$= 6k^2$
3	2	$3k \times 2k$	$= 6k^2$
4	1	$4k \times k$	$= 4k^2$

$20k^2 = 20(0.1)^2 = 0.2$

$P(X_1 + X_2 = 8) = P(X_1 = 4 \cap X_2 = 4) = 4k \times 4k = 16k^2$

$= 16(0.1)^2 = 0.16$

e) $P(1.5 < X_1 + X_2 < 3.5) = P(X_1 + X_2 = 2) + P(X_1 + X_2 = 3)$

$= 0.01 + 0.04 = 0.05$

2\.
- Fixed number of trials, n
- Trials are independent, fixed p
- Can be viewed as having 2 outcomes

X = no. wait over ½ hour

$X \sim Bin(8, 0.3)$

List on calc

X	P
0	0.0576
1	0.1976
2	0.2964
3	0.2541
⋮	smaller probabilities

highest probability

$\Rightarrow$ 2 is most probable