Statistics 10 – Binomial Distribution 2 – Solutions

Section 1

1. Area rectangle $= 2.4 \times 2 = 4.8$

i.e. 4.8cm^2 represents 8

$\times 11.125$ $\times 11.125$

53.4cm^2 represents 89

2. $\bar{x} = \frac{\Sigma x}{n} = \frac{920}{200} = 4.6$

$s.d_x = \sqrt{\frac{\Sigma x^2}{n} - \bar{x}^2} = \sqrt{\frac{5032}{200} - 4.6^2} = 2$

3. a) $n = 50$ $\frac{n}{2} = \frac{50}{2} = 25$

$2+7+9 = 18$

$2+7+9+14 = 32$

18th — 25th — 32nd

104.5 — 109.5

$104.5 + \left(\frac{25-18}{32-18}\right) \times 5 = 107 = \text{Median}$ (5 = Class width)

b) $\frac{n}{4} = 12.5$

$2+7 = 9$

$2+7+9 = 18$

9th — 12.5th — 18th

99.5 — 104.5

$99.5 + \left(\frac{12.5-9}{18-9}\right) \times 5 = 101.44$ (5 = Class width)

c) $\frac{30n}{100} = 15$

9th — 15th — 18th

99.5 — 104.5

$99.5 + \left(\frac{15-9}{18-9}\right) \times 5 = 102.83$

Section 2

1. $X \sim Bin(10, 0.3)$

a) $P(X \leq 3) = 0.6496$

b) $P(X > 7) = 1 - P(X \leq 7) = 1 - 0.9984 = 0.0016$

c) $P(2 \leq X \leq 5) = P(X = 2,3,4,5) = P(X \leq 5) - P(X \leq 1)$

$= 0.9527 - 0.1493 = 0.8034$

d) $P(X=3) + P(X=8) = 0.2668 + 0.001447 = 0.2682$

e) $P(X > 3 \wedge X < 8) = P(3 < X < 8) = P(X = 4, 5, 6, 7)$

$= P(X \leq 7) - P(X \leq 3)$

$= 0.9984 - 0.6496 = 0.3488$

(9)

2. X = no. tails $X \sim Bin(3, p)$

a) $P(X=1) = {}^3C_1\, p^1(1-p)^2 = 3p(1-p)^2$

b) $P(X=2) = {}^3C_2\, p^2(1-p)^1 = 3p^2(1-p)$

c) $P(X=1) = 2P(X=2) \Rightarrow 3p(1-p)^2 = 6p^2(1-p)$

$\div 3p(1-p) \Rightarrow 1-p = 2p$

$\Rightarrow 1 = 3p$

$\Rightarrow p = \frac{1}{3}$

(6)

3. B = no. with blue pen $B \sim Bin(12, 0.6)$

a) $P(B=6) = 0.1766$

b) $P(B=9) = 0.1419$

c) $P(6 \leq B \leq 9) = P(B = 6,7,8,9) = P(B \leq 9) - P(B \leq 5)$

$= 0.9165 - 0.1582 = 0.7583$ (8)

4. X = no. days with ≥ 7 oktas. $X \sim Bin(15, 0.2)$

a) $P(X \geq 3) = 1 - P(X \leq 2) = 1 - 0.3980 = 0.6020$

b) $P(6 < X < 14) = P(X = 7, 8, 9, \ldots 13) = P(X \leq 13) - P(X \leq 6) = 1.000 - 0.9819$

$= 0.0181$

c) $P(X=4) = 0.1876$

(6)

5. X = no. students with perfect attendance

$X \sim Bin(10, 0.65)$

a) $P(X < 4) = P(X \leq 3) = 0.0256$ ✓

b) $P(X > 7) = 1 - P(X \leq 7) = 1 - 0.7383 = 0.2617$ ✓

c) $P(5 < X \leq 8) = P(X = 6, 7, 8) = P(X \leq 8) - P(X \leq 5)$ ✓

$= 0.914 - 0.2485 = 0.6655$ ✓

d) 2 or more missing at least one lesson ⇒ 8 or less have perfect attendance ✓

$P(X \leq 8) = 0.914$ ✓

e) One student not having perfect attendance (illness) might affect chance of another not having perfect attendance ✓ (contagious).

i.e. X is unlikely to be independent ✓

Or p unlikely to be fixed because as some student more susceptible to illness / absence than others.

(10)

6. I = no. infected $I \sim Bin(20, 0.7)$

a) $P(I > 15) = 1 - P(I \leq 15) = 1 - 0.7626 = 0.2374$ ✓

b) $P(I < 10) = P(I \leq 9) = 0.0171$ ✓

(4)

Total = 40

Section 3

D = no. defectives

A: $D_A \sim Bin(10, 0.01)$

$P(D_A = 0) = 0.9044$

$P(D_A = 1) = 0.0914$

$P(\text{Accepted}) = P(D_A = 0) + P(D_A = 1) \times P(D_A = 0)$

$= 0.9044 + 0.0914 \times 0.9044$

$= 0.9044 + 0.0826 = 0.987$

B: $D_B \sim Bin(20, 0.01)$

$P(\text{Accepted}) = P(D_B \leq 1) = 0.983$

$0.987 > 0.983 \Rightarrow$ Should use method A