

Statistics 13 - Hypothesis Testing 2 - Solutions

Section 2

1. a) The critical value is the boundary of the critical region.
 If H_1 is $p > \dots$ critical value is smallest value in critical region.
 If H_1 is $p < \dots$ critical value is largest value in critical region.
 b) Critical region is range of values in which H_1 is accepted.
 c) Acceptance region is range of values in which H_0 is accepted.

2. a) $H_0: p = 0.2$
 $H_1: p > 0.2$

$X \sim \text{Bin}(10, 0.2)$

95% \rightarrow

H_0	H_1	5%
c		

 $P(X \geq c) = 0.05$
 $\Rightarrow 1 - P(X \leq c-1) = 0.05$
 $\Rightarrow P(X \leq c-1) = 0.95$

From calc $P(X \leq 3) = 0.8791$
 $P(X \leq 4) = 0.9672$
 $c-1$ between 3 and 4
 $\Rightarrow c$ between 4 and 5

i.e. Critical region is $x \geq 5$
 b) $x \geq 5 \Rightarrow$ Accept H_0 , Reject H_1

3. $H_0: p = 0.15$
 $H_1: p < 0.15$
 $X \sim \text{Bin}(20, 0.15)$

5% \rightarrow

H_1	H_0	95%
c		

 $P(X \leq c) = 0.05$
 $P(X \leq 0) = 0.0387$
 $P(X \leq 1) = 0.1755$
 c between 0 and 1

i.e. Critical region is $x \leq 0$ (i.e. $x=0$)
 \Rightarrow Acceptance region $x \geq 1$

Section 1

1. a)

x	1	2	3	4
$P(X=x)$	$\frac{k}{25}$	$\frac{k}{25}$	$\frac{k}{25}$	$\frac{k}{25}$

$k + \frac{k}{25} + \frac{k}{25} + \frac{k}{25} = 1 \Rightarrow \frac{25k}{25} = 1 \Rightarrow k = \frac{12}{25}$

b)

x	1	2	3	4
$P(X=x)$	$\frac{10}{25}$	$\frac{6}{25}$	$\frac{4}{25}$	$\frac{3}{25}$

c) i) $P(X > 2) = \frac{4}{25} + \frac{3}{25} = \frac{7}{25}$

ii) $P(1 < X < 4) = \frac{6}{25} + \frac{4}{25} = \frac{10}{25} = \frac{2}{5}$

iii) $P(X > 4) = 0$

2. a)

	S	S'
T	0.4	
T'	0.18	0.6
	0.3	0.7

 $P(S \cap T) = 0.3 \times 0.18 = 0.054$
 $P(S \cap T') = 0.3 \times 0.4 = 0.12$
 $P(S \cap T) = P(S \cap T')$
 $\Rightarrow S, T$ are independent

b) i) $P(S \cup T) = 1 - P(S' \cap T')$
 $= 1 - (0.6 \times 0.18) = 1 - 0.108 = 0.892$

ii) $P(S' \cup T) = 1 - P(S \cap T')$
 $= 1 - 0.18 = 0.82$

3. a)

x	1	2	3	4
$P(X=x)$	$\frac{k}{25}$	$\frac{k}{25}$	$\frac{k}{25}$	$\frac{k}{25}$

$k + k + k + k + k = 1 \Rightarrow 5k = 1 \Rightarrow k = \frac{1}{5}$

b) $P(X > 1) = k + k + k + k = 4k = 7 \times \frac{1}{5} = \frac{7}{5}$

c) $1\% < 5\% \Rightarrow$ If reject H_0 at 5% would also reject at 1%
 \Rightarrow will not be different

d) Expect $np = 20 \times 0.4 = 8$. Therefore would clearly be in acceptance region if $x = 8$. (6)

6 a) $X = n$, students late $X \sim \text{Bin}(20, 0.1)$

$H_0: p = 0.1$
 $H_1: p > 0.1$

0.99	0.01
H_0	H_1

c	

 $P(X \geq c) = 0.01$
 $\Rightarrow P(X \leq c-1) = 0.99$
 From calc $P(X \leq 5) = 0.988$ } $c-1$ between 5 and 6
 $P(X \leq 6) = 0.9976$ } $\Rightarrow c$ between 6 and 7

i.e. critical region is $x \geq 7 \Rightarrow 7$ is smallest number
 b) X unlikely to follow a binomial distribution as not independent
 (one person being late makes it more likely others will be late) (4)

7. $X = n$, seeds germinating $X \sim \text{Bin}(10, 0.85)$

$H_0: p = 0.85$
 $H_1: p < 0.85$

0.05	0.95
H_1	H_0

c	

 $P(X \leq c) = 0.05$
 From calc $P(X \leq 6) = 0.0499$ } c between 6 and 7
 $P(X \leq 7) = 0.1798$

i.e. critical region is $x \leq 6$ (3)

Total = 25

$X \sim \text{Bin}(6, 0.3)$

$H_0: p = 0.3$
 $H_1: p > 0.3$

0.95	0.05
H_0	H_1

c	

 $P(X \geq c) = 0.05$
 $P(X \leq c-1) = 0.95$
 $P(X \leq 2) = 0.9295$ } $c-1$ between 3 and 4
 $P(X \leq 3) = 0.989$ } c between 4 and 5

i.e. critical region is $x \geq 5$

4. $X = n$, spaces available
 $X \sim \text{Bin}(10, 0.35)$
 Z college weeks = 2.5 days

$H_0: p = 0.35$
 $H_1: p > 0.35$

0.99	0.01
H_0	H_1

c	

 $P(X \geq c) = 0.01$
 $\Rightarrow P(X \leq c-1) = 0.99$
 From calc $P(X \leq 6) = 0.9739$ } $c-1$ between 6 and 7
 $P(X \leq 7) = 0.9951$ } $\Rightarrow c$ between 7 and 8

i.e. Critical region is $x \geq 8$ (3)

5. $X = n$, A grades

a) $X \sim \text{Bin}(20, 0.4)$

$H_0: p = 0.4$
 $H_1: p \neq 0.4$

$P(X \leq c_1) = 0.025$

0.025	0.95	0.025
H_1	H_0	H_1

c_1 c_2		

 $P(X \leq 3) = 0.0159$ } c_1 between 3 and 4
 $P(X \leq 4) = 0.0509$

$P(X \geq c_2) = 0.025$

$\Rightarrow P(X \leq c_2-1) = 0.975$
 $P(X \leq 11) = 0.9434$ } c_2-1 between 11 and 12
 $P(X \leq 12) = 0.9789$ } $\Rightarrow c_2$ between 12 and 13

i.e. critical regions are $x \leq 3$ and $x \geq 13$ (3)

b) $x = 11$ in acceptance region \Rightarrow Accept H_0 , Reject H_1

i.e. insufficient evidence that proportion of students achieving A's has changed