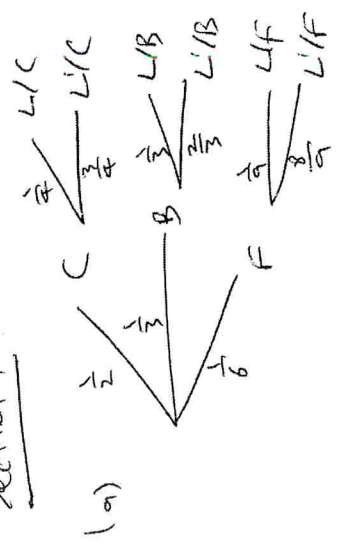


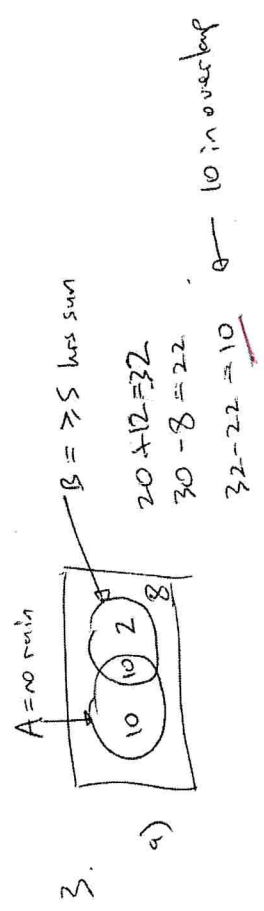
Statistics 3 - Normal Distribution - Solutions

Section 1



- b) $P(FNL) = \frac{1}{6} \times \frac{1}{4} = \frac{1}{24}$
 - i) $P(L') = P(CNL') + P(BNL') + P(FNL')$
 $= \frac{1}{2} \times \frac{2}{4} + \frac{1}{3} \times \frac{2}{3} + \frac{1}{6} \times \frac{2}{3}$
 $= \frac{2}{8} + \frac{2}{9} + \frac{2}{27} = \frac{161}{216} = \frac{0.7454}{1} = 0.7454$
 - iii) $P(F/L) = \frac{P(CNL) + P(BNL) + P(FNL)}{P(L)}$
 $= \frac{\frac{1}{2} \times \frac{1}{4} + \frac{1}{3} \times \frac{1}{3} + \frac{1}{6} \times \frac{1}{3}}{1 - 0.7454}$
 $= \frac{\frac{17}{72}}{\frac{55}{216}} = \frac{51}{55} = 0.927$
2.

X	1	2	3	4	5
P(X)	k	4k	9k	4k	5k
- a) $k + 4k + 9k + 4k + 5k = 1 \Rightarrow 23k = 1 \Rightarrow k = \frac{1}{23}$
 - b) $P(X > 2.4) = P(X = 3, 4, 5) = 9k + 4k + 5k = 18k = \frac{18}{23}$
 - c) $Y = \text{no. times } X > 2.4 \quad Y \sim \text{Bin}(15, \frac{18}{23})$
 $P(Y \geq 3) = 1 - P(Y \leq 2) = 1 - 0.6643 = 0.3357$



- b) $P(A') = 1 - \frac{20}{30} = \frac{1}{3}$
- i) $P(A \cup B') = \frac{10 + 10 + 8}{30} = \frac{14}{15}$
- ii) $P(A'/B) = \frac{2}{12} = \frac{1}{6}$
- c) $P(A \cup B')$ means either no rain, or less than 5 hours of sun (or both)
- $P(A'/B)$ means Given it had at least 5 hours of sun, find the probability it rained
- d) $X = \text{no. days rained} \quad X \sim \text{Bin}(30, \frac{1}{3})$
 $P(X=11) = {}^{30}C_{11} (\frac{1}{3})^{11} (\frac{2}{3})^{19} = 0.1391$
 Assumed probability of rain in 1997 and 1987 is the same
 Also assumed it is independent.
 Both of these assumptions are very unlikely to be true - not reasonable
- e) $P(A|B)$ has no effect on $P(A)$ then $P(A|B) = P(A)$
 Alternatively $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A) \times P(B)}{P(B)} = P(A)$
- b) A is not affected by B happening, A is also unaffected by B not happening
 $P(A|B) = P(A)$
 $P(A|B') = P(A)$

c) $P(A|B) = P(B) - P(A \cap B) = P(B) - P(A)P(B) = P(B)(1 - P(A)) = P(B)P(A')$
 Alternatively if A has no effect on B, then A not happening can have no effect on B either.

d) If A and B were mutually exclusive A happening would make it impossible for B to happen, i.e. not independent. Since A and B are independent they can not be mutually exclusive.

e) If A and B are exhaustive, A not happening would make it certain for B to happen, i.e. not independent. Since A and B are independent they can not be exhaustive.

Section 2

1. $X \sim N(20, 3^2)$
 a) 95% between 2 s.d. i.e. between 20-2(3) and 20+2(3) \Rightarrow between 14 and 26

b) inflection at $\mu \pm \sigma$ i.e. 23 and 17

2. $X \sim N(50, 6^2)$

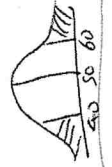
a) i) $P(X < 60) = P(-100 < X < 60) = 0.9522$

ii) $P(X > 40) = P(40 < X < 1000) = 0.9522$

iii) $P(42 < X < 55) = 0.7065$

iv) $P(45 < X < 58) = 0.7065$

b) i) ii) due to symmetry of graph about 50



3. a) $X \sim N(1017, 8^2)$ $\sigma^2 = 64 \Rightarrow \sigma = 8$

b) i) $P(X > 1025) = P(1025 < X < 2000) = 0.1587$

ii) $P(X < 1000) = P(0 < X < 1000) = 0.0168$

iii) $1 - P(990 < X < 1000) = 1 - 0.0164 = 0.9836$

4. $X = \text{time (mins)}$

$X \sim N(12\frac{1}{2}, (\frac{1}{3})^2)$

a) $P(X < 11) = P(0 < X < 11) = 0.1908$

34 runners \Rightarrow expect $34 \times 0.1908 = 6.49$

i.e. 6 runners

b) Assumed times are roughly symmetric, with 95% of data within 2 standard deviations of the mean.

5. $X \sim N(15, 2^2)$

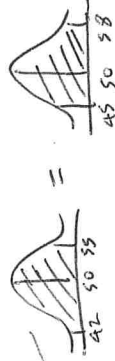
a) $P(X > 18) = P(18 < X < 100) = 0.0668$

b) $Y = \text{no. values above 18}$

$Y \sim \text{Bin}(10, 0.0668)$

i) $P(Y = 2) = 0.1155$

ii) $P(Y < 2) = P(Y \leq 1) = 0.8599$



6

6. X = weight of coffee

Y = weight of glass

$$X \sim N(101.5, 7.5^2)$$

$$Y \sim N(269, 5.5^2)$$

$$P(X < 97)$$

$$P(Y < 259)$$

$$= P(0 < X < 97) =$$

$$= P(0 < Y < 259)$$

$$= 0.2743$$

$$= 0.0345$$

$$P(X < 97 \cap Y < 259) = 0.2743 \times 0.0345 = 0.00945$$

3

7. $X \sim N(15, 3^2)$

a) i) $P(X < 11.5) = P(-100 < X < 11.5) = 0.1217$

ii) $P(X > 12.5) = P(12.5 < X < 100) = 0.7977$

iii) $P(11.5 < X < 12.5) = 0.0807$

d) would expect $P(X < 11.5) + P(X > 12.5) + P(11.5 < X < 12.5) = 1$

(in this case they add to 1.0001 due to rounding)

e) Because X is continuous $P(X = 12) \neq P(11.5 < X < 12.5)$

for all continuous data the probability that it is

an exact value is 0 i.e. $P(X = 12) = 0$.

Charles is incorrect.

5

Total = 30

