

# Statistics 4 - Inverse Normal Distribution - Solutions

## Section 1

1. a)  $P(X < Q_1) = \frac{1}{4}$

b)  $P(X > Q_2) = \frac{1}{2}$

c) i)  $Y = n_0$  values below  $Q_3$

$Y \sim \text{Bin}(4, \frac{3}{4})$

$P(Y=4) = 0.3164$

ii)  $Z = n_0$  values between  $Q_1$  and  $Q_2$

$Z \sim \text{Bin}(4, \frac{1}{4})$

$P(Z \geq 2) = 1 - P(Z \leq 1) = 1 - 0.7383 = 0.2617$

2.  $X \sim N(70, 8^2)$

a)  $P(X < 60) = P(0 < X < 60) = 0.1056$

b)  $P(X > 80) = P(80 < X < 100) = 0.1056$

c)  $P(X = 55) = 0$

d)  $P(X < 62 \cup X > 78) = 1 - P(62 < X < 78) = 1 - 0.6822 = 0.3178$

3. a)  $(X \text{ is continuous data})$

Median & Mean are very close

$n = 30 + 31 = 61$

Normal  $\Rightarrow$  68% of data within one s.d. of mean

$\mu = 1018 \quad \sigma \approx 10 \Rightarrow P(1008 < X < 1028) = 0.68$

b)  $X \sim N(1018, 10.025^2)$

$P(1015 < X < 1025) = 0.3751$

4.  $X = n_0$  phones with cracked screen

$X \sim \text{Bin}(100, 0.1)$

$H_0: p = 0.1$

$H_1: p < 0.1$

$\frac{10\%}{H_1} \mid \frac{90\%}{H_0}$

$P(X \leq 0) = 0.0147$   
 $P(X \leq 1) = 0.0804$   
 $P(X \leq 2) = 0.2228$

$\} c$  between 1 and 2

Initially  $n < c$   
 Then  $n+1 > c \Rightarrow n=1$

5.  $X \sim N(\mu, \sigma^2)$

a) i)   $P(X < a) > 0.5 \Rightarrow a > \mu$

ii)   $P(X < a) < 0.5 \Rightarrow a < \mu$

iii)   $P(X > a) > 0.5 \Rightarrow a < \mu$

iv)   $P(X > a) < 0.5 \Rightarrow a > \mu$

b)  $a = \mu \Rightarrow P(X < a) = P(X > a) = 0.5$

Section 2

1.a) Continuous data  
Data normally symmetrical with modal group in middle

b)  $X \sim N(47.5, 10^2)$

i)  $P(35 < X < 45) = 0.2956$   
 $n=100 \Rightarrow$  Predict  $0.2956 \times 100 = 29.56 \approx 30$  plums

ii)  $P(m < 25) = P(-100 < m < 25) = 0.0122$   
 $n=100 \Rightarrow$  Predict  $0.0122 \times 100 = 1.22 \approx 1$  plum

c)  $35 \leq m < 45$   $f_{req} = 29$   $m_{med} = 30$  } Both answers very close to reality  $\Rightarrow$   $m < 25$   $f_{req} = 0$   $m_{med} = 1$  }  $m_{med}$  is suitable

d) This is not advisable. Plum crops likely to vary from year to year. (will probably still be normally distributed)

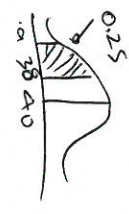
2.  $X \sim N(40, 10^2)$

a)  $q = 31.58$

b)  $P(X > q) = 0.7 \Rightarrow P(X < q) = 0.3 \Rightarrow q = 34.76$

c)  $P(X < M) = 0.5 \Rightarrow P(X < a) = 0.6 \Rightarrow a = 42.53$

d)  $P(X < 38) = P(-100 < X < 38) = 0.4207$   
 $P(9 < X < 38) = 0.25 \Rightarrow P(X < 9) = 0.4207 - 0.25 = 0.1707 \Rightarrow a = 30.49$



3.  $X \sim N(50, 25) \Rightarrow X \sim N(50, 5^2)$

a) Median  $\approx$  Mean = 50

b) Mode = Mean = 50

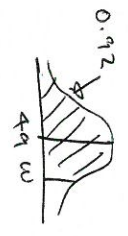
c)  $P(X < LQ) = 0.25 \Rightarrow LQ = 46.63$

d)  $P(X < UQ) = 0.75 \Rightarrow UQ = 53.37$

4.  $X =$  weight (kg)  
 $X \sim N(49, 3^2)$

a)  $P(X > 55) = P(55 < X < 100) = 0.0228$

b)  $P(X > w) = 0.92 \Rightarrow P(X < w) = 0.08 \Rightarrow w = 44.28$



c)  $Y =$  no. bags that weigh above 55 kg  
 $Y \sim \text{Bin}(5, 0.0228)$

i)  $P(Y=1) = 0.1040$

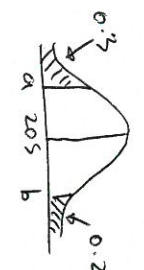
ii)  $P(Y=0) = 0.8911$   
 $X =$  weight average (g)

$Y \sim N(205, 25^2)$

a) i)  $P(X < 250) = P(-100 < X < 250) = 0.9641$

ii)  $P(200 < X < 250) = 0.5433$

b)  $P(X < a) = 0.3 \Rightarrow a = 191.9$   
 $P(X > b) = 0.2 \Rightarrow P(X < b) = 0.8 \Rightarrow b = 226.0g$



c)  $P$  not fixed (with fixed replacement)  
 Charles ... thinks to then to sell smaller oranges