Statistics 5 - Standardised Normal Distribution

Section 1

1. $X \sim N(30, 4^2)$

a) $P(20 < X < 38) = 0.9710$

b) $P(X < 25) = P(-100 < X < 25) = 0.1056$

c) $P(X > 20) = P(20 < X < 200) = 0.9938$

2. X = Weight (g), $X \sim N(500, 20^2)$

a) $P(X < 480) = P(0 < X < 480) = 0.1587$

b) $P(X < 510) = P(0 < X < 510) = 0.6915$

c) $P(X > 450) = P(450 < X < 800) = 0.9938$

3. X = IQ score, $X \sim N(100, 15^2)$

a) $P(X > 130) = P(130 < X < 300) = 0.0228$

b) Y = no. people with IQ above 130

$Y \sim Bin(20, 0.0228)$

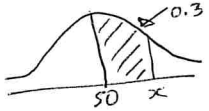
$P(Y \geq 3) = 1 - P(Y \leq 2) = 1 - 0.9899 = 0.0101$

4. $X \sim N(50, 8^2)$

a) $P(X < x) = 0.3 \Rightarrow x = 45.8$

b) $P(X > x) = 0.2 \Rightarrow P(X < x) = 0.8 \rightarrow x = 56.7$

c) $P(50 < X < x) = 0.3$

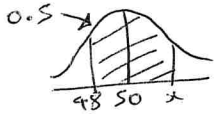


$\Rightarrow$ $P(X < 50) = 0.5$

$\Rightarrow P(X < x) = 0.5 + 0.3 = 0.8$

$\Rightarrow x = 56.7$

d) $P(48 < X < x) = 0.5$



$P(X < 48) = P(0 < X < 48) = 0.401$

$\Rightarrow P(X < x) = 0.5 + 0.401 = 0.901$

$\Rightarrow x = 60.3$

Section 2

1. $X \sim N(80, 10^2)$

a) i) $Y = X - 80 \Rightarrow Y$ mean $= 80 - 80 = 0$ ✓

Y s.d $= 10$ ✓

ii) $Z = \frac{Y}{10} \Rightarrow Z$ mean $= \frac{0}{10} = 0$ ✓

Z s.d $= \frac{10}{10} = 1$ ✓

b) i) $Y \sim N(0, 10^2)$ ✓

ii) $Z \sim N(0, 1^2)$ ✓

c) $z = \frac{x - \mu}{\sigma}$ ✓ (7)

2. $Z \sim N(0, 1^2)$

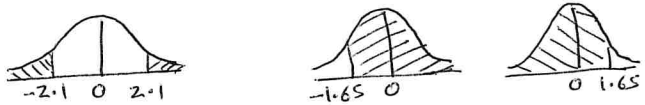
a) $P(Z > 2.1) = P(2.1 < Z < 20) = 0.0179$ ✓

b) $P(Z < 1.65) = P(-10 < Z < 1.65) = 0.9505$ ✓

c) $P(Z > -1.65) = P(-1.65 < Z < 10) = 0.9505$ ✓

d) $P(Z < -2.1) = P(-10 < Z < -2.1) = 0.0179$ ✓

e)



✓ (6)

Because Z is symmetrical about 0, and $P(Z > a) = P(Z < -a)$

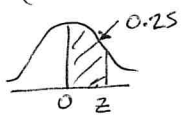
$P(Z < a) = P(Z > -a)$

3. $Z \sim N(0, 1^2)$

a) $P(Z < z) = 0.3 \Rightarrow z = -0.52$ ✓

b) $P(Z > z) = 0.9 \Rightarrow P(Z < z) = 0.1 \Rightarrow z = -1.28$ ✓

c) $P(0 < Z < z) = 0.25$

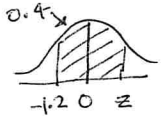


$P(Z < 0) = 0.5$ ✓

$\Rightarrow P(Z < z) = 0.5 + 0.25 = 0.75$

$\Rightarrow z = 0.67$ ✓

d) $P(-1.2 < Z < z) = 0.4$



$P(Z < -1.2) = P(-10 < z < -1.2) = 0.1151$ ✓

$\Rightarrow P(Z < z) = 0.4 + 0.1151 = 0.5151$

$\Rightarrow z = 0.04$ ✓

(6)

4. a) i) $P(Z > z) = 0.01 \Rightarrow z = 2.3263$ ✓

ii) $P(Z > z) = 0.3 \Rightarrow z = 0.5244$ ✓

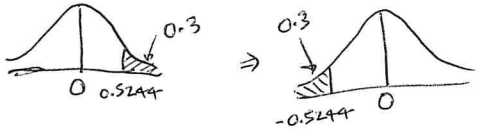
iii) $P(Z > z) = 0.15 \Rightarrow z = 1.0364$ ✓

b) i)

$P(Z < z) = 0.01$

$\Rightarrow z = -2.3263$ ✓

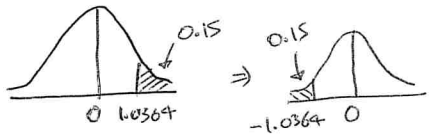
ii)



$P(Z < z) = 0.3$

$\Rightarrow z = -0.5244$ ✓

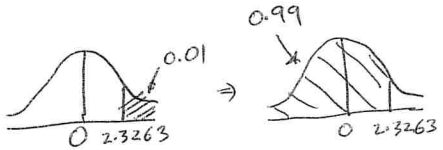
iii)



$P(Z < z) = 0.15$

$\Rightarrow z = -1.0364$ ✓

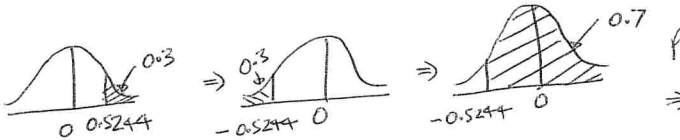
iv)



$P(Z < z) = 0.99$

$\Rightarrow z = 2.3263$ ✓

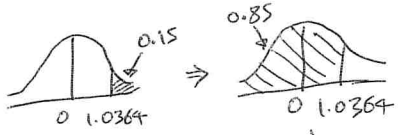
v)



$P(Z > z) = 0.7$

$\Rightarrow z = -0.5244$ ✓

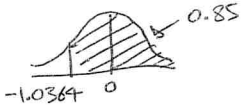
vi)



$P(Z < z) = 0.85$

$\Rightarrow z = 1.0364$ ✓

vii)

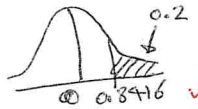


$P(Z > z) = 0.85$

$\Rightarrow z = -1.0364$ ✓

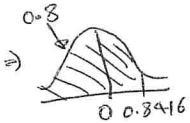
(10)

5. a) From table $P(Z > 0.8416) = 0.2 \Rightarrow$



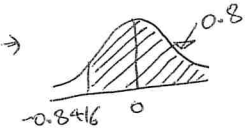
✓

i)



$P(Z < z) = 0.8 \Rightarrow z = 0.8416$ ✓

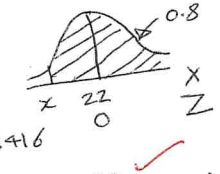
ii)



$P(Z > z) = 0.8 \Rightarrow z = -0.8416$ ✓

b) X = length of banana. $X \sim N(22, 1.5^2)$

$P(X > x) = 0.8 \Rightarrow$



$z = \frac{x - \mu}{\sigma} \Rightarrow -0.8416 = \frac{x - 22}{1.5}$ ✓ $\Rightarrow x - 22 = -0.8416 \times 1.5$ ✓

$\Rightarrow x = -1.2624 + 22$ ✓

$\Rightarrow x = 20.74$ (2dp)

(6)