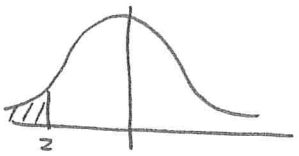
Statistics 6 - Normal dist. (finding μ & σ)

Section 1

1a, From tables $z = 1.96$, checking sketch 

$\therefore$ $z = -1.96$

b, $X \sim N(70, 4.5^2)$ $\quad z = \frac{x - \mu}{\sigma}$

$-1.96 = \frac{x - 70}{4.5}$, $x = 61.18$

2, $r = 0.8253$ (to 4dp) from Calculator

Hypp test; $H_0 : \rho = 0$ $\quad n = 5$, 5%, 1 tailed test

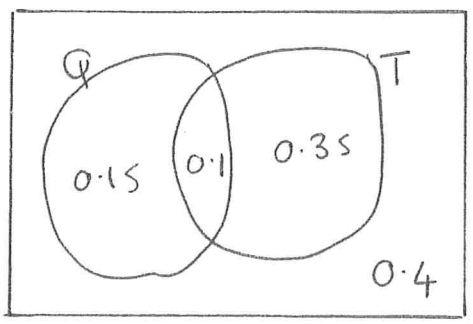
$H_1 : \rho > 0$ $\quad$ Critical value = 0.8054

$0.8253 > 0.8054$ $\therefore$ Reject H_0

There is not enough evidence to show that there is no correlation.

3, a, 10%

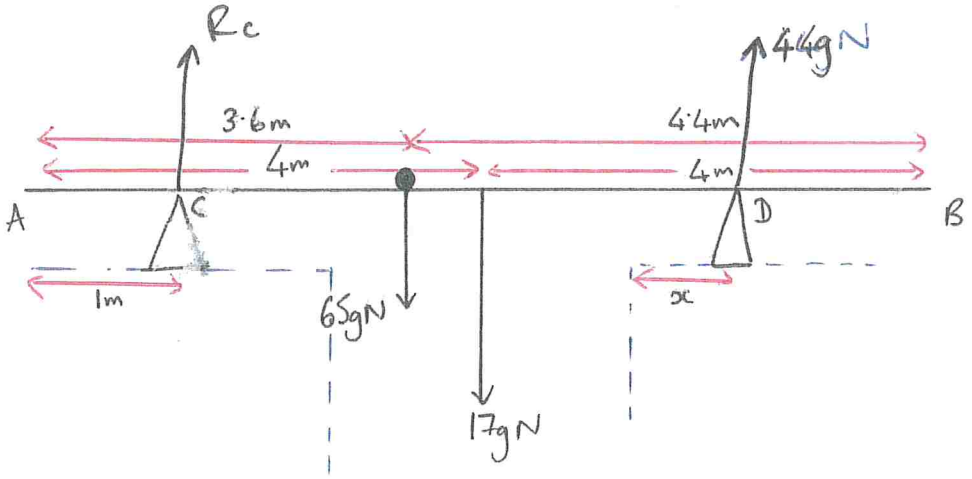
b,



c, $P(\text{Q only} \mid \text{reads quality}) = \frac{0.15}{0.6}$

$= \frac{15}{60}$ 0.25 or $\frac{1}{4}$

4, a,



b, Resolving moments about C

$$17g \times 3 + 65g \times 2.6 = 44g \times (4.6 + x)$$

$$220g = (202.4 + 44x)g$$

$$44x = 17.6, \quad x = 0.4m$$

c, Plank is uniform ; Weight is acting in centre of the plank

d, Plank is a rod ; We can ignore shape and size of plank.

Section 2

1a, A statistical model is a way of simplifying a real world problem, it can be used to make predictions about a real word problem.

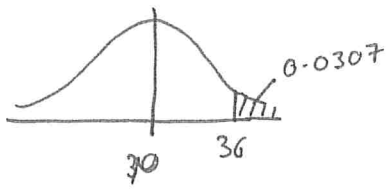
b, i, Normal distribution can be be used ; data is continuous and likely to be symmetrical.

ii, Not a Normal distribution ; Data is discrete.

2, $X \sim N(\mu, 7^2)$ $X = 24$ $p = 0.8106$ $z = 0.8801$

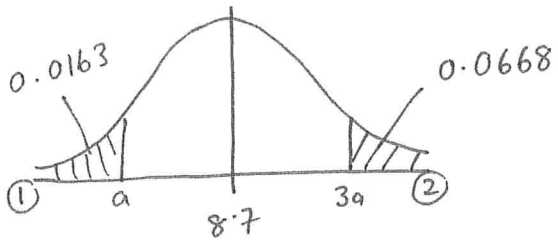
$$0.8801 = \frac{24 - \mu}{7} \qquad \mu = 17.84$$

3, $Y \sim N(30, \sigma^2)$ $\quad p = 0.9693$ $\quad (1 - 0.0307)$ $\quad Y = 36$ $\quad z = 1.871$



$1.871 = \frac{36 - 30}{\sigma}$, $\sigma = 3.21$ ✓ ✓

4,



① $p = 0.0163$ $\quad X = a$, $\quad z = -2.137$

$-2.137 = \frac{a - 8.7}{\sigma}$ ✓

① $-2.137\sigma = a - 8.7$ $\quad \times 3$

② $p = 0.9332$ $\quad (1 - 0.0668)$, $\quad X = 3a$, $\quad z = 1.5$

$1.5 = \frac{3a - 8.7}{\sigma}$ ✓

② $1.5\sigma = 3a - 8.7$ →

① $-6.411\sigma = 3a - 26.1$ Sim eq ✓

② $1.5\sigma = 3a - 8.7$ $\quad -$

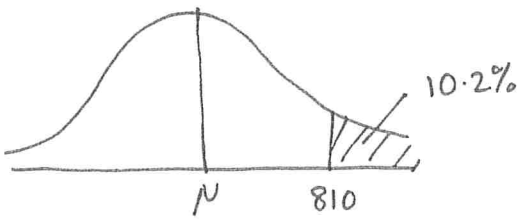
$-7.911\sigma = -17.4$

$\sigma = 2.2$ ✓

Sub $\sigma = 2.2$ into ① $\quad -2.137 \times 2.2 = a - 8.7$

$a = 4$ ✓

5, $X \sim N(\mu, 18^2)$



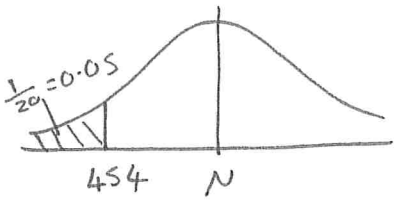
$p = 0.898$ $\quad (1 - 0.102)$, $\quad X = 810$, $\quad z = 1.27$

$1.27 = \frac{810 - \mu}{18}$ ✓

$\mu = 787.14$

$\mu = 787$ ✓

6, $X \sim N(\mu, 1.6^2)$ $p = 0.05$, $x = 454$, $z = -1.645$ ✓



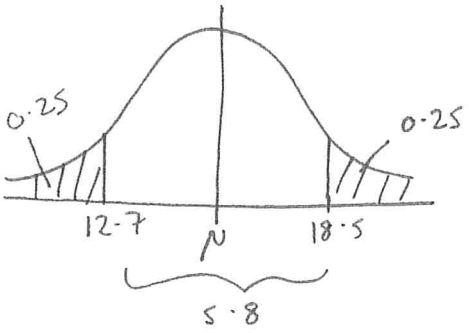
$$-1.645 = \frac{454 - \mu}{1.6}$$ ✓

$\mu = \cancel{461.567}$ ~~546~~ 456.632

$\mu = 461.6$ g ✓ accuracy to 4 sig fig.

$\mu = 456.6$ g

7,



a, Upper quartile $= 12.7 + 5.8$

$= 18.5$ ✓

b, By symmetry $\mu \Rightarrow 12.7 + \frac{1}{2}(5.8)$ ✓

$= 15.6$ ✓

$X \sim N(15.6, \sigma^2)$

$p = 0.25$, $X = 12.7$, $z = -0.674$

$$-0.674 = \frac{12.7 - 15.6}{\sigma}$$ ✓

✓

$$\sigma = \frac{-2.9}{-0.674} = 4.303$$ ✓

$\therefore$ variance, $\sigma^2 = 18.5$ (to 3sf)

Other methods (simultaneous equations) are ok too!

c, Model $\Rightarrow \mu = 15.6$, $\sigma = 4.303$

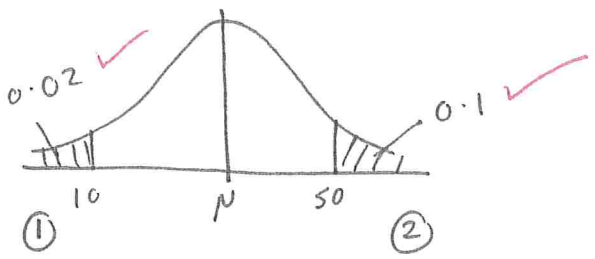
Observed $\Rightarrow \mu = 15.3$, $\sigma^2 = 20.1$

$\sigma = 4.48$ ✓

I would say the model is ✓ quite suitable, the s.d is slightly too small, but still close.

8,

a,



b, ① $p = 0.02$, $x = 10$, $z = -2.05$

$$-2.05 = \frac{10 - \mu}{\sigma}$$

② $p = 0.9$, $x = 50$

$z = 1.28$

$$1.28 = \frac{50 - \mu}{\sigma}$$

① $-2.05\sigma = 10 - \mu$

② $1.28\sigma = 50 - \mu$

$-2.05\sigma = 10 - \mu$ −

$3.33\sigma = 40$

$\sigma = 12.01$

$\sigma = 12.0$ (to 3sf)

sub into ② $1.28 \times 12.01 = 50 - \mu$

$\mu = 34.6$

Total marks = 34