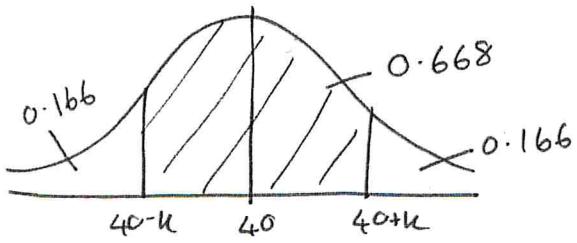
Solutions - Normal distribution (approx. binomial)

Section 1

1, $X \sim N(40, 9)$

$\sigma = 3$



$1 - 0.668 = 0.332$

$\div 2 = 0.166$

$p = 0.166 \rightarrow X = 37.09$

So $40 - k = 37.09$

$k = 2.91$

2, $x = t$, $y = \log d$

$y = -0.785 + 0.123x$

$\log d = -0.785 + 0.123t$

$d = 10^{-0.785 + 0.123t}$

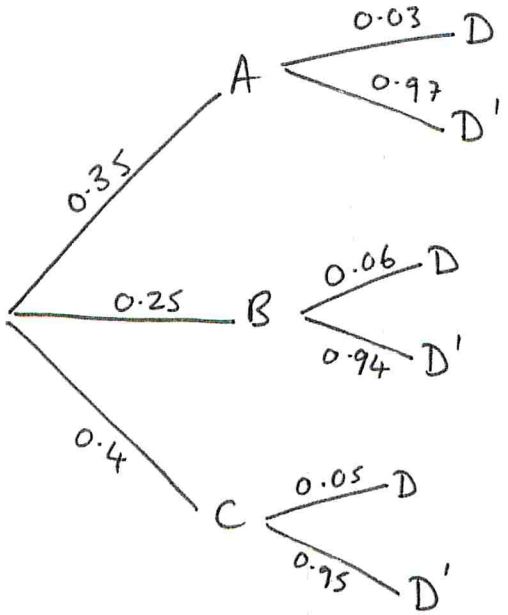
$= 10^{-0.785} \times 10^{0.123t}$

$d = 0.164 \times 1.33^t$

So $a = 0.164$ $b = 1.33$

3,

a,



b, i) $P(A \cap D) = 0.35 \times 0.03$

$= 0.0105$

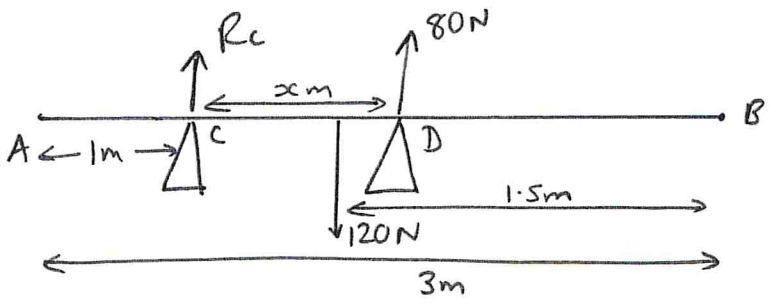
ii) $P(\text{defective}) = 0.35 \times 0.03 + 0.25 \times 0.06 + 0.4 \times 0.05$

$= 0.0455$

3c, $P(C \mid D) = \frac{P(C \cap D)}{P(D)} = \frac{0.4 \times 0.05}{0.0455}$

$= 0.4396$

4,



a, Resolving moments about C

$0.5 \times 120 = 80 \times x$

$60 = 80x$

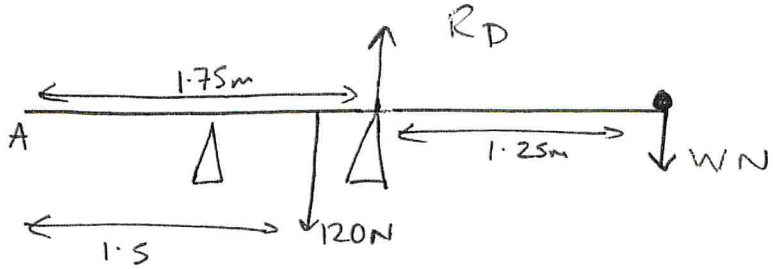
$x = 0.75m$

Resolving forces

$R_c + 80 = 120$

$R_c = 40N$

b, If tilting about D, then $R_c = 0$



Moments about D

$1.25W = 0.25 \times 120$

$W = 24N$

c, Resolve forces $R_D = 24 + 120 = 144N$.

d, If it is a particle, size and shape can be ignored and weight acts at R exactly

Section 2

1, a, $X \sim B(150, 0.6) \rightarrow X \approx N(90, 36)$ ✓

$\sigma = 6$ ✓

n is large

p is close to 0.5 ✓

b, $X \sim B(30, 0.4)$

n is too small (< 50) ✓

c, $X \sim B(250, 0.8)$ p is not close to 0.5 ✓

d, $X \sim B(800, 0.47) \rightarrow X \approx N(376, 199.28)$ ✓

$\sigma = 14.12$ ✓

n is large, p close to 0.5 ✓

2, $Y \sim B(120, 0.54) \rightarrow Y \approx N(64.8, 29.808)$ ✓

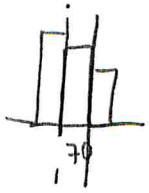
$\sigma = 5.46$

a, $P(Y \leq 55) \rightarrow P(Y < 55.5) = 0.044$ ✓

continuity correction

b, $P(Y > 70) \rightarrow P(Y > 70.5) = 0.148$ ✓

C.C.



c, $P(60 < Y \leq 75) = P(Y < 75.5) - P(Y < 60.5)$

$Y > 60$

$= 0.97498 - 0.21548 = 0.7595$

3, $X \sim B(250, 0.4) \longrightarrow X \approx N(100, 60)$

$\sigma = 7.75$

a, $P(90 < X \leq 100) \longrightarrow P(X < 100.5) - P(X < 90.5)$ ✓ ✓

$= \underline{0.4156}$ ✓

b, $P(X = 130) = P(129.5 < X < 130.5)$ ✓ ✓

$= \underline{0.000029} \quad = (2.9 \times 10^{-5})$ ✓

4, X is no. of female students. $X \sim B(100, 0.58)$

$X \approx N(58, 24.36)$ ✓

$\sigma = 4.94$ ✓

a, $P(X > 50) = P(X > 50.5)$ ✓

c.c.

$= \underline{0.9355}$ ✓

b, from Binomial $X \sim B(100, 0.58)$ ✓

$P(X > 50) = 1 - P(X \leq 50)$ ✓

$= 1 - 0.06498$

$= 0.93502$ ✓

$\%$ error $= \dfrac{\text{approx} - \text{exact}}{\text{exact}} \times 100$

$= \dfrac{0.9355 - 0.93502}{0.93502} \times 100 = \underline{0.051\%}$ ✓ ✓

5, a, $p = 0.5$ ✓

b, If $p = 0.5$, the probs of Binomial dist will be completely symmetrical in the same way that the normal dist is symmetrical.

c, $X \sim B(200, 0.48) \longrightarrow X \approx N(96, 49.92)$ ✓

$\sigma = 7.065$ ✓

$P(90 \leq X < 105) \longrightarrow P(89.5 < X < 104.5)$ ✓ ✓

$= 0.7068$ ✓

6, $X \sim B(8, 0.3)$ ✓

a, $P(X = 2) = 0.2965$ ✓ ('from Calc')

b, $P(X > 4) = 1 - P(X \leq 4)$ ✓

$= 1 - 0.9420$

$= 0.058$ ✓

c, $X \sim B(150, 0.3) \longrightarrow X \approx N(45, 31.5)$ ✓

$\sigma = 5.61$ ✓

$P(X > 50) \longrightarrow P(X > 50.5)$

c.c. ✓

$= 0.1634$ ✓

7, $X \sim B(12, 0.45)$ ✓ X is no. of white flowers.

a, $P(X = 5) = \underline{0.2225}$ ✓

b, $P(X > 6) = 1 - P(X \leq 6) = 1 - 0.7393$

$= \underline{0.2607}$ ✓

/ ✓
More than
6 plants with white flowers.

c, 10 batches Y is no. of batches with more white than coloured plants.

p = more plants with white flowers = 0.2607

$Y \sim B(10, 0.2607)$ ✓

$P(Y = 3) = \underline{0.2567}$ ✓

d, $X \sim B(50, 0.45) \longrightarrow X \approx N(22.5, 12.375)$ ✓

$\sigma = 3.52$ ✓

$P(X > 25) \Rightarrow P(X > 25.5)$ ✓

$= \underline{0.1970}$ ✓

Total marks = 56