**9MA0/02: Pure Mathematics Paper 2 Mark scheme**

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| **Question** | **Scheme** | **Marks** | **AOs** |
| **1** |    | M1 | 1.1a |
|   | A1 | 1.1b |
|  length of minor arc   | dM1 | 3.1a |
|   | A1 | 1.1b |
|  | **(4)** |  |
| **1****Alt** |    | M1 | 1.1a |
|   | A1 | 1.1b |
|  length of major arc   |  |  |
|  length of minor arc   | dM1 | 3.1a |
|   | A1 | 1.1b |
|  | **(4)** |  |
| **(4 marks)** |
| **Question 1 Notes:** |
| **M1:** | Applies formula for the area of a sector with  i.e.  with **Note:** Allow M1 for considering ratios. E.g.   |
| **A1:** | Uses a correct equation  to obtain a radius of 7.5 |
| **dM1:** | Depends on the previous M mark. A complete process for finding the length of the minor arc *AB*, by either*
*
 |
| **A1:** | Correct exact answer in its simplest form, e.g.  |

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| **Question** | **Scheme** | **Marks** | **AOs** |
| **2(a)** | Attempts to substitute into either  | M1 | 1.1b |
|  |  |  |
|   | M1 | 1.1b |
|   |  |  |
|   \* | A1\* | 2.1 |
|  |  | **(3)** |  |
| **(b)(i)** | E.g.* Adele is working in degrees and not radians
* Adele should substitute  and not  into the approximation
 | B1 | 2.3 |
| **(b)(ii)** | , so  gives a good approximation. | B1 | 2.4 |
|  | **(2)** |  |
| **(5 marks)** |
| **Question 2 Notes:** |
| **(a)(i)** |  |
| **M1:** | See scheme |
| **M1:** | Substitutes  into  and attempts to apply **Note:** It is not a requirement for this mark to write or refer to the term in   |
| **A1\*:** | Correct proof with no errors seen in working. **Note:** It is not a requirement for this mark to write or refer to the term in  |
| **(a)(ii)** |  |
| **B1:** | See scheme |
| **(b)(i)** |  |
| **B1:** | See scheme |
| **(b)(ii)** |  |
| **B1:** | Substitutes  into  to give awrt 7.962 ***and*** an appropriate conclusion. |

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| **Question** | **Scheme** | **Marks** | **AOs** |
| **3 (a)** |   | B1 | 3.3 |
|  | **(1)** |  |
| **(b)** |  | M1 | 3.4 |
|  minutes (1 dp) | A1 | 1.1b |
|  | **(2)** |  |
| **(c)** | A valid evaluation of the model, which relates to the large values of *t*. E.g.* As  then the model is not true for large values of *t*
* does not have any solutions and so the model predicts that tea in the room will never be  So the model does not work for large values of *t*
* *t* = 120 $⇒$ *θ* = 25 + 50*e* −0.03(120) = 26.36… which is not approximately equal to 20.3, so the model is not true for large values of *t*
 | B1 | 3.5a |
|  | **(1)** |  |
| **(4 marks)** |
| **Question 3 Notes:** |
| **(a)** |  |
| **B1:** | Applies  to give the complete model  |
| **(b)** |  |
| **M1:** | Applies  and their value of *A* to the model and rearranges to make  the subject. |
|  | **Note:** Later working can imply this mark. |
| **A1** | Obtains 11.9 (minutes) with no errors in manipulation seen. |
| **(c)** |  |
| **B1** | See scheme  |

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| **Question** | **Scheme** | **Marks** | **AOs** |
| **4(a)** | 5*x**y**O* | Correct graph in quadrant 1 and quadrant 2 with V on the *x*-axis | B1 | 1.1b |
| States  and **or** marked in the correct position on the *x‑*axis **and** 5 marked in the correct position on the *y*-axis  | B1 | 1.1b |
|  | **(2)** |  |
| **(b)** |   |  |  |
|   **and**  | M1 | 1.1b |
|  {critical values are }   | A1 | 1.1b |
|  | **(2)** |  |
| **(c)** |  |  |  |
| E.g.* Solves  to give

and solves  to also give * Sketches graphs of  and .

Indicates that these graphs meet at the point   | M1 | 3.1a |
| Hence using set notation, e.g. *
*
*
 | A1 | 2.5 |
|  |  | **(2)** |  |
| **(6 marks)** |

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| **Question 4 Notes:** |
| **(a)** |  |
| **B1:** | See scheme |
| **B1:** | See scheme |
| **(b)** |  |
| **M1:** | See scheme |
| **A1:** | Correct answer, e.g.*
*
*
 |
| **(c)** |  |
| **M1:** | A complete process of finding that  and  meet at ***only*** one point.This can be achieved either algebraically or graphically. |
| **A1:** | See scheme.**Note:** Final answer must be expressed using set notation. |

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| **Question** | **Scheme** | **Marks** | **AOs** |
| **5** |   intersects  at two distinct points |  |  |
| Eliminate *y* and forms quadratic equation = 0 or quadratic expression   | M1 | 3.1a |
|   | A1 | 1.1b |
|   | dM1 | 2.1 |
|   |  |  |
|  Critical value obtained of  | B1 | 1.1b |
|   o.e. | A1 | 1.1b |
|  | **(5)** |  |
| **5****Alt 1** | Eliminate *y* and forms quadratic equation = 0 or quadratic expression   | M1 | 3.1a |
|  |  |  |
|  | A1 | 1.1b |
|   | dM1 | 2.1 |
|  |  |  |
|  Critical value obtained of  | B1 | 1.1b |
|   o.e. | A1 | 1.1b |
|  | **(5)** |  |
| **5****Alt 2** |  . So  | M1 | 3.1a |
| A1 | 1.1b |
|  | dM1 | 2.1 |
|  Critical value obtained of  | B1 | 1.1b |
|   o.e. | A1 | 1.1b |
|  | **(5)** |  |
| **(5 marks)** |

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| **Question 5 Notes:** |
| **M1:** | Complete strategy of eliminating *x* or *y* and manipulating the resulting equation to form a quadratic equation = 0 or a quadratic expression  |
| **A1:** | Correct algebra leading to either * or

or a one-sided quadratic of either  or *

or a one-sided quadratic of e.g.  |
| **dM1:** | Depends on the previous M mark.Interprets  intersecting  at two distinct points by applying  to their quadratic equation or one-sided quadratic. |
| **B1:** | See scheme |
| **A1:** | Correct answer, e.g.*
*
 |
| **Alt 2** |  |
| **M1:** | Complete strategy of using differentiation to find the values of *x* and *y* where  is a tangent to  |
| **A1:** | Correct algebra leading to   |
| **dM1:** | Depends on the previous M mark.Full method of substituting their  into *l* and attempting to find the value for *k*. |
| **B1:** | See scheme |
| **A1:** | Deduces correct answer, e.g.*
*
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| **Question** | **Scheme** | **Marks** | **AOs** |
| **6(a)** |  |  |  |
| Crosses *x*-axis  |  |  |
| *x* coordinates are 1 and 8  | B1 | 1.1b |
|  | **(1)** |  |
| **(b)** | Complete strategy of setting  and rearranges to make   | M1 | 3.1a |
|  |  |  |
|  | M1 | 1.1b |
| A1 | 1.1b |
|  \* | A1\* | 2.1 |
|  | **(4)** |  |
| **(c)** | Evaluates both  and  | M1 | 1.1b |
|  and Sign change and as  is continuous, the *x* coordinate of *Q* lies between  and  | A1 | 2.4 |
|  | **(2)** |  |
| **(d)(i)** |  | B1 | 1.1b |
| **(d)(ii)** |  (2 dp) | B1 | 2.2a |
|  | **(2)** |  |
| **(9 marks)** |

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| **Question 6 Notes:** |
| **(a)** |  |
| **B1:** | Either * 1 and 8
* on Figure 2, marks 1 next to *A* and 8 next to *B*
 |
| **(b)** |  |
| **M1:** | Recognises that *Q* is a stationary point (and not a root) and applies a complete strategy of setting  and rearranges to make  |
| **M1:** | Applies , where **Note:** This mark can be recovered for work in part (c) |
| **A1:** |  or equivalent**Note:** This mark can be recovered for work in part (c) |
| **A1\*:** | Correct proof with no errors seen in working.  |
| **(c)** |  |
| **M1:** | Evaluates both  and  |
| **A1:** |  and  or  (truncated) **and** a correct conclusion |
| **(d)(i)** |  |
| **B1:** | See scheme |
| **(d)(ii)** |  |
| **B1:** | Deduces (e.g. by the use of further iterations) that the *x* coordinate of *Q* is 3.54 accurate to 2 dp |
|  | **Note:**  |

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| **Question** | **Scheme** | **Marks** | **AOs** |
| **7(a)** |   | B1 | 3.3 |
|  | M1 | 1.1b |
|   | A1 | 1.1b |
|  \* | A1 \* | 2.1 |
|  | **(4)** |  |
| **(b)** |  and evidence of understanding that either *
* vertical intercept  or
 | M1 | 2.1 |
|   | A1 | 1.1b |
| vertical intercept   | A1 | 1.1b |
|  | **(3)** |  |
| **(c)** | e.g.*
*
 | B1 | 2.2a |
|  which can be implied by  | B1 | 1.1b |
|  | **(2)** |  |
| **(d)(i)** | Initial area (i.e. ) of bacterial culture that was first placed onto the circular dish. | B1 | 3.4 |
| **(d)(ii)** | E.g.* Rate of increase per hour of the area of bacterial culture
* The area of bacterial culture increases by “15%” each hour
 | B1 | 3.4 |
|  |  | **(2)** |  |
| **(e)** | The model predicts that the area of the bacteria culture will increase indefinitely, but the size of the circular dish will be a constraint on this area. | B1 | 3.5b |
|  | **(1)** |  |
| **(12 marks)** |

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| **Question 7 Notes:** |
| **(a)** |  |
| **B1:** | Translates the scientist’s statement regarding proportionality into a differential equation, which involves a constant of proportionality. e.g.  |
| **M1:** | Correct method of separating the variables *p* and *t* in their differential equation |
| **A1:** |  with or without a constant of integration |
| **A1\*:** | Correct proof with no errors seen in working.  |
| **(b)** |  |
| **M1:** | See scheme |
| **A1:** | Correctly finds   |
| **A1:** | Correctly finds   |
| **(c)** |  |
| **B1:** | Uses algebra to correctly deduce either * from
* from
 |
| **B1:** | See scheme |
| **(d)(i)** |  |
| **B1:** | See scheme |
| **(d)(ii)** |  |
| **B1:** | See scheme |
| **(e)** |  |
| **B1:** | Gives a correct long-term limitation of the model for *p*. (See scheme). |

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| **Question** | **Scheme** | **Marks** | **AOs** |
| **8(a)** | ,  |  |  |
|   | M1 | 1.1b |
| A1 | 1.1b |
|  | M1 | 3.1a |
| When  | dM1 | 3.4 |
|  | A1 | 1.1b |
|  | **(5)** |  |
| **(b)** |  | M1 | 3.4 |
|   | A1 | 1.1b |
|  | **(2)** |  |
| **(7 marks)** |
| **Question 8 Notes:** |
| **(a)** |  |
| **M1:** | Differentiates *V* with respect to *h* to give  |
| **A1:** |  |
| **M1:** | Attempts to solve the problem by applying a complete method of  |
| **M1:** | Depends on the previous M mark. |
|  | Substitutes  into their model for  which is in the form  |
| **A1:** | Obtains the correct answer 0.4  |
| **(b)** |  |
| **M1:** | Realises that rate for of for  has no effect when the rate is increased to for  and so substitutes  into their model for  which is in the form  |
| **A1:** | Obtains the correct answer 0.5  |

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| **Question** | **Scheme** | **Marks** | **AOs** |
| **9(a)** | E.g. midpoint *PQ*  | M1 | 1.1b |
|   which is the centre point *A*,  so *PQ* is the diameter of the circle. | A1 | 2.1 |
|  | **(2)** |  |
| **(a)****Alt 1** |   | M1 | 1.1b |
|  So so *PQ* is the diameter of the circle. | A1 | 2.1 |
|  | **(2)** |  |
| **(a)****Alt 2** | **and either** *
*
 | M1 | 1.1b |
| e.g. as , then *PQ* is the diameter of the circle. | A1 | 2.1 |
|  | **(2)** |  |
| **(b)** | Uses Pythagoras in a correct method to find either the radius or diameter of the circle. | M1 | 1.1b |
|  | M1 | 1.1b |
| A1 | 1.1b |
|  | **(3)** |  |
| **(c)** | Distance  or  | M1 | 3.1a |
|   | A1 | 1.1b |
|  | **(2)** |  |
| **(d)** |  or  | M1 | 3.1a |
|  (to 0.1 of a degree) | A1 | 1.1b |
|  | **(2)** |  |
| **(9 marks)** |

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| **Question 9 Notes:** |
| **(a)** |  |
| **M1:** | Uses a correct method to find the midpoint of the line segment *PQ* |
| **A1:** | Completes proof by obtaining  and gives a correct conclusion. |
| **(a)** |  |
| **Alt 1** |  |
| **M1:** | Full attempt to find the equation of the line *PQ* |
| **A1:** | Completes proof by showing that  lies on *PQ* and gives a correct conclusion. |
| **(a)** |  |
| **Alt 2** |  |
| **M1:** | Attempts to find distance *PQ* and either one of distance *AP* or distance *AQ* |
| **A1:** | Correctly shows either * , supported by  and gives a correct conclusion
* , supported by  and gives a correct conclusion
 |
| **(b)** |  |
| **M1:** | **Either*** uses Pythagoras correctly in order to find the **radius**. Must clearly be identified as the **radius**. E.g.  or  or  or

**or*** uses Pythagoras correctly in order to find the **diameter**. Must clearly be identified as the **diameter**. E.g.  or
 |
|  | **Note:** This mark can be implied by just 30 clearly seen as the **diameter** or 15 clearly seen as the **radius** (may be seen or implied in their circle equation) |
| **M1:** | Writes down a circle equation in the form   |
| **A1:** |  or  or  |
| **(c)** |  |
| **M1:** | Attempts to solve the problem by using the circle property “the perpendicular from the centre to a chord bisects the chord” and so applies Pythagoras to write down an expression of the form . |
| **A1:** |  by correct solution only |
| **(d)** |  |
| **M1:** | Attempts to solve the problem by e.g. using the circle property “the angle in a semi-circle is a right angle” and writes down either  or **Note:** Also allow   |
| **A1:** | 41.8 by correct solution only |

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| **Question** | **Scheme** | **Marks** | **AOs** |
| **10 (a)** |   | B1 | 2.2a |
|  | **(1)** |  |
| **(b)** | Attempts to apply   | M1 | 3.1a |
|   | A1 | 1.1b |
|   | M1 | 3.1a |
|  | A1 | 1.1b |
|  | M1 | 1.1b |
| A1 | 1.1b |
|  | M1 | 2.2a |
|   |  |  |
|   \* | A1\* | 2.1 |
|  | **(8)** |  |
| **(b)****Alt 1** | Attempts to apply , with a substitution of  | M1 | 3.1a |
|   | A1 | 1.1b |
|   | M1 | 3.1a |
|  | A1 | 1.1b |
|  | M1 | 1.1b |
| A1 | 1.1b |
|  | M1 | 2.2a |
|   |  |  |
|   \* | A1 \* | 2.1 |
|  | **(8)** |  |
| **(9 marks)** |

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| **Question** | **Scheme** | **Marks** | **AOs** |
| **10 (b)****Alt 2** | Attempts to apply , with a substitution of  | M1 | 3.1a |
|   | A1 | 1.1b |
|   | M1 | 3.1a |
|  | A1 | 1.1b |
|  | M1 | 1.1b |
| A1 | 1.1b |
|  | M1 | 2.2a |
|   |  |  |
|   \* | A1 \* | 2.1 |
|  | **(8)** |  |

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| **Question 10 Notes:** |
| **(a)** |  |
| **B1:** | Uses  with  to deduce the correct domain,  |
| **(b)** |  |
| **M1:** | Attempts to solve the problem by either * a parametric process or
* a Cartesian process with a substitution of either  or
 |
| **A1:** | Obtains * from a parametric approach
* from a Cartesian approach with
* from a Cartesian approach with
 |
| **M1:** | Applies a strategy of attempting to express either ,  or  as partial fractions |
| **A1:** | Correct partial fractions for their method |
| **M1:** | Integrates to give either *
* where
* where
 |
| **A1:** | Correct integration for their method |
| **M1:** | Either * Parametric approach: Deduces and applies limits of 2 and 0 in *t* and subtracts the correct way round
* Cartesian approach: Deduces and applies limits of 3 and 1 in *u*, where  and subtracts the correct way round
* Cartesian approach: Deduces and applies limits of 4 and 2 in *v*, where  and subtracts the correct way round
 |
| **A1\*:** | Correctly shows that the area of *R* is , with no errors seen in their working |

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| **Question** | **Scheme** | **Marks** | **AOs** |
| **11** | Arithmetic sequence,  |  |  |
|   | M1 | 2.1 |
|    | A1 | 1.1b |
| . So   | M1 | 2.2a |
|  | M1 | 1.1b |
|   which is a square number | A1 | 2.1 |
|  | **(5)** |  |
| **(5 marks)** |
| **Question 11 Notes:** |
| **M1:** | Complete method to find the value of *k* |
| **A1:** | Uses a correct method to find  |
| **M1:** | Uses their value of *k* to deduce the common difference and the first term  of the arithmetic series. |
| **M1:** | Applies  with their  and their *d*. |
| **A1:** | Correctly shows that the sum of the series is  and makes an appropriate conclusion. |

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| **Question** | **Scheme** | **Marks** | **AOs** |
| **12** | Complete process to find at least one set of coordinates for *P*. The process must include evidence of* differentiating
* setting  to find
* substituting  into  to find
 | M1 | 3.1a |
|  | B1 | 1.1b |
| Applies   | M1 | 2.2a |
| giving at least one of either  or  | A1 | 1.1b |
|  | M1 | 1.1b |
| So in specified range, , by cso | A1 | 1.1b |
|  has no solutions, and so there are exactly 2 possible points *P*. | B1 | 2.1 |
|  | **(7)** |  |
| **(7 marks)** |
| **Question 12 Notes:** |
| **M1:** | See scheme |
| **B1:** | Correct differentiated equation. E.g.  |
| **M1:** | Uses the information “the tangent to *C* at the point *P* is parallel to the *x*-axis” to deduce and apply  and finds   |
| **A1:** | See scheme |
| **M1:** | For substituting one of their values from  into  and so finds   |
| **A1:** | Selects coordinates for *P* on *C* satisfying  and   |
|  | i.e. finds  and no other points by correct solution only |
| **B1:** | Complete argument to show that there are exactly 2 possible points *P*. |

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| **Question** | **Scheme** | **Marks** | **AOs** |
| **13(a)** |  |  |  |
|  | M1 | 1.2 |
|   | M1 | 1.1b |
|   | M1 | 2.1 |
| A1 | 1.1b |
|   | A1\* | 2.1 |
|  | **(5)** |  |
| **(b)** | ;  |  |  |
|  | M1 | 2.2a |
|  | M1 | 1.1b |
| A1 | 1.1b |
|  | M1 | 2.1 |
|  | A1 | 1.1b |
|  | **(5)** |  |
| **(10 marks)** |

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| **Question 13 Notes:** |
| **(a)** |  |
| **M1:** | Writes  and  |
| **M1:** | Combines into a single fraction with a common denominator |
| **M1:** | Applies  to the denominator **and** applies either |
|  | *
* and
* and

to the numerator and manipulates to give a one term numerator expression |
| **A1:** | Correct algebra leading to  or equivalent. |
| **A1\*:** | Correct proof with correct notation and no errors seen in working |
| **(b)** |  |
| **M1:** | Uses the result in part (a) in an attempt to deduce either  or  and uses  to write down or imply  |
| **M1:** | Applies  or  |
|  | and attempts to solve  to give   |
| **A1:** | Uses a correct method to obtain  |
| **M1:** | Uses  in a complete method to find the second solution,   |
| **A1:** | Uses a correct method to obtain , with no extra solutions given either inside or outside the required range   |

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| **Question** | **Scheme** | **Marks** | **AOs** |
| **14 (i)** | For an explanation or statement to show when the claim  failsThis could be e.g.* when   or  is not greater than or equal to
* when   or  is not greater than or equal to
 | M1 | 2.3 |
| followed by an explanation or statement to show when the claim  is true. This could be e.g.* or 9 is greater than or equal to 4
* when

and a correct conclusion. E.g.* so the claim  is sometimes true
 | A1 | 2.4 |
|  | **(2)** |  |
| **(ii)** | Assume that  is a rational number So , where *p* and *q* integers,  and the HCF of *p* and *q* is 1  | M1 | 2.1 |
|   is divisible by 3 and so *p* is divisible by 3 | M1 | 1.1b |
| A1 | 2.2a |
| So where *c* is an integerFrom earlier,  | M1 | 2.1 |
|   is divisible by 3 and so *q* is divisible by 3 | A1 | 1.1b |
| As both *p* and *q* are both divisible by 3 then the HCF of *p* and *q* is not 1This contradiction implies that  is an irrational number | A1 | 2.4 |
|  | **(6)** |  |
| **(8 marks)** |

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| **Question 14 Notes:** |
| **(i)** |  |
| **M1:** | See scheme |
| **A1:** | See scheme |
| **(ii)** |  |
| **M1:** | Uses a method of proof by contradiction by initially assuming that  is rational and expresses  |
|  |  in the form , where *p* and *q* are correctly defined. |
| **M1:** | Writes  and rearranges to make  the subject |
| **A1:** | Uses a logical argument to prove that *p* is divisible by 3  |
| **M1:** | Uses the result that *p* is divisible by 3, (to construct the initial stage of proving that  is also divisible by 3), by substituting  into their expression for  |
| **A1:** | Hence uses a correct argument, in the same way as before, to deduce that *q* is also divisible by 3 |
| **A1:** | Completes the argument (as detailed on the scheme) that  is irrational. |
|  | **Note:** All the previous 5 marks need to be scored in order to obtain the final A mark. |