Pearson Edexcel

Mark Scheme

Mock Paper (Set 2)

December 2019

Pearson Edexcel GCE Mathematics

Pure Mathematics 1 Paper 9MA0/01

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 100.

2. The Edexcel Mathematics mark schemes use the following types of marks:
   - **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
   - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - **B** marks are unconditional accuracy marks (independent of M marks)
   - Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- The second mark is dependent on gaining the first mark

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses <u>and indicates which response they wish to submit</u>, examiners should mark this response.
   If there are several attempts at a question <u>which have not been crossed out</u>, examiners should mark the final answer which is the answer that is the <u>most complete</u>.

6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

General Principles for Further Pure Mathematics Marking

(*But note that specific mark schemes may sometimes override these general principles*)

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2+bx+c)=(x+p)(x+q)$, where $|pq|=|c|$, leading to $x=\ldots$

$(ax^2+bx+c)=(mx+p)(nx+q)$, where $|pq|=|c|$ and $|mn|=|a|$, leading to $x=\ldots$

2. Formula

Attempt to use the correct formula (with values for *a*, *b* and *c*)

3. Completing the square

Solving $x^2+bx+c=0$: $\left(x\pm\frac{b}{2}\right)^2\pm q\pm c=0,\quad q\neq 0$, leading to $x=\ldots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. $(x^n \to x^{n-1})$

2. Integration

Power of at least one term increased by 1. $(x^n \to x^{n+1})$

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Question	Scheme	Marks	AOs
1(a)	$93\,\text{m}^2$	B1	3.4
		(1)	
(b)	$40 = 105 - 12\text{e}^{0.08t} \Rightarrow 12\text{e}^{0.08t} = 65$	M1	3.1b
	$\Rightarrow 0.08t = \ln\left(\frac{65}{12}\right) \Rightarrow t = ...$	dM1	1.1b
	21.1 days	A1	1.1b
		(3)	
(c)	Substitutes $t = 30$ into $A = 105 - 12\text{e}^{0.08t} \Rightarrow A = ...$	M1	3.4
	$A = -27.3$ and states that Stuart cannot use the model as it gives a negative area	A1	2.4
		(2)	
			(6 marks)

Notes:

(a)

B1: $93\,\text{m}^2$. This requires the units

(b)

M1: For using the model with $A = 40$ and proceeding to $P\text{e}^{0.08t} = Q$

dM1: For correct use of lns and proceeding to a value for t

A1: Accept awrt 21.1 days or $t = 21.1$

(c)

M1: Uses the model in an attempt to find A when $t = 30$

A1: Finds $A = -27$ at $t = 30$ and states the area of weed cannot be negative

Question	Scheme	Marks	AOs
2 (a)	$(1+4x)^{\frac{1}{2}} = 1+\frac{1}{2}\times 4x + \frac{\frac{1}{2}\times -\frac{1}{2}}{2!}\times(4x)^2 + \frac{\frac{1}{2}\times -\frac{1}{2}\times -\frac{3}{2}}{3!}\times(4x)^3$	M1 A1	1.1b 1.1b
	$= \underline{1+2x} - 2x^2 + 4x^3 + ...$	<u>B1</u> A1	1.1b 1.1b
		(4)	
(b)	The expansion is not valid if $\lvert x\rvert > \frac{1}{4}$ $\frac{25}{4} > \frac{1}{4}$ so should not be used	B1	2.4
		(1)	
(c)	Substitutes $x = \frac{1}{100}$ into $(1+4x)^{\frac{1}{2}}$ gives $\frac{\sqrt{26}}{5}$	M1	1.1b
	Explains that $x = \frac{1}{100}$ is substituted into $1+2x-2x^2+4x^3$ and you multiply the result by 5	A1	2.4
		(2)	
			(7 marks)

Notes:

(a)

M1: Attempts the binomial expansion with $n = \frac{1}{2}$ to get the correct structure for term 3 or term 4.

For example look, for term 3, a form $\frac{\frac{1}{2}\times -\frac{1}{2}}{2!}\times(*x)^2$

A1: Correct (unsimplied) term3 and term 4

B1: $1+2x$

A1: $-2x^2+4x^3$

(b)

B1: For a correct explanation as to why $x = \frac{25}{4}$ should not be used. The explanation must reference the $\frac{1}{4}$ and not just state that it is too big.

(c)

M1: Substitutes $x = \frac{1}{100}$ into $(1+4x)^{\frac{1}{2}}$ gives $\frac{\sqrt{26}}{5}$

A1: Requires a full (and correct) explanation as to how the expansion can be used to estimate $\sqrt{26}$

Question	Scheme	Marks	AOs
3 (a)	Uses the sequence formula $u_{n+1} = \dfrac{4}{2-u_n}$ once $u_2 = 4$	M1	1.1b
	$(u_1 = 1), u_2 = 4, u_3 = -2, u_4 = 1$	A1	1.1b
	Explains that since $u_1 = u_4$ then sequence is periodic with period 3	A1	2.4
		(3)	
(b)	$\sum_{n=1}^{50} u_n = 16\times(1+4+-2)+4+1$	M1	3.1a
	$= 53$	A1	1.1b
		(2)	
			(5 marks)

Notes:

(a)

M1: Applies the sequence formula $u_{n+1} = \dfrac{4}{2-u_n}$ seen once

A1: $u_2 = 4, u_3 = -2, u_4 = 1$. There is no need to see either u_1 or any of the labels. Look for the correct terms in the correct order.

A1: Explains that since $u_1 = u_4$ then sequence is periodic with period 3

(b)

M1: Uses a clear strategy to find the sum to 50 terms. This will usually be found using multiples of the first three terms.

For example you may see $\sum_{n=1}^{50} u_n = \left(\sum_{n=1}^{48} u_n\right) + u_{49} + u_{50} = 16\times(1+4+-2)+4+1$

$$\sum_{n=1}^{50} u_n = \left(\sum_{n=1}^{51} u_n\right) - u_{51} = 17\times(1+4+-2)-(-2)$$

A1: 53

Question	Scheme	Marks	AOs
4(a)	Sets up identity $4x^3-19x^2+28x-4=(Ax+B)(x-2)^2+C$ **And** finds values of A, B or C	M1	2.1
	For two of $A=4, B=-3, C=8$	A1	1.1b
	For all three of $A=4, B=-3, C=8$	A1	1.1b
		(3)	
(b)	$\int \mathrm{h}(x)\,\mathrm{d}x = \int 4x-3+\frac{8}{(x-2)^2}\,\mathrm{d}x$		
	$=2x^2-3x-\frac{8}{x-2}+c$	M1 M1 A1ft	1.1b 1.1b 1.1b
		(3)	
			(6 marks)

Notes:

(a)

M1: For showing clear calculations and algebraic reasoning leading to values of A, B or C.

E.g. If an identity is used then it must be correct. Sets $4x^3-19x^2+28x-4=(Ax+B)(x-2)^2+C$ and finds values of A, B and C are found by substituting or equating terms

E.g. If division is used then $(x-2)^2 \to x^2 \pm 4x \pm 4$ and the division must lead to a linear quotient of

$4x+B$ with a remainder that is independent of x. $x^2-4x+4 \overline{\smash{\big)}\,4x^3-19x^2+28x-4}$ with quotient $4x^2+....$

(Note: This method would not be expected but is an acceptable way to score the marks)

A1: For two of $A=4, B=-3, C=8$

A1: For all three of $A=4, B=-3, C=8$

(b)

M1: For a correct attempt at integrating either the $Ax+B$ term or the $\frac{C}{(x-2)^2}$ term

M1: For a correct attempt at integrating both the $Ax+B$ term and the $\frac{C}{(x-2)^2}$ term

A1ft: $2x^2-3x-\frac{8}{x-2}+c$ but follow through on their non-zero values of A, B and C.

Question	Scheme	Marks	AOs
5(a)	Uses $\left\lvert\overrightarrow{AB}\right\rvert = 5\sqrt{2} \Rightarrow 5^2 + (p-3)^2 = \left(5\sqrt{2}\right)^2$	M1	1.1b
	Solves to find at least one value for $p \Rightarrow (p-3)^2 = 25 \Rightarrow p = ...$	M1	1.1b
	$p = -2, 8$	A1	2.1
		(3)	
(b)	$\overrightarrow{AC} = (q+2)\mathbf{i} + 4\mathbf{j}$	M1	1.1b
	Uses $\tan\left(\frac{\pi}{3}\right) = \frac{4}{q+2}$ with $\tan\left(\frac{\pi}{3}\right) = \sqrt{3}$ to find q	dM1	2.1
	$q = \frac{4}{\sqrt{3}} - 2$ or $\frac{4\sqrt{3}}{3} - 2$ or other exact form	A1	1.1b
		(3)	
	(6 marks)		

Notes:

(a)

M1: Uses Pythagoras' theorem in an attempt to form an equation in *p*. There must be an attempt to subtract the coordinates to find $\overrightarrow{AB}$

M1: Uses a correct method to find at least one value of *p* from their quadratic equation

A1: Uses all steps correctly to find $p = -2, 8$

(b)

M1: Attempts to finds $\overrightarrow{AC}$ by subtracting components

dM1: Uses correct trigonometry to set up an equation in *q*. To score this mark $\tan\left(\frac{\pi}{3}\right) = \sqrt{3}$ or equivalent must be used

A1: $q = \frac{4}{\sqrt{3}} - 2$ or $\frac{4\sqrt{3}}{3} - 2$ or other exact form

Question	Scheme	Marks	AOs
6 (a)	$\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$	B1	1.1a
	$\tan 3\theta = \dfrac{\tan 2\theta + \tan\theta}{1-\tan 2\theta\tan\theta} = \dfrac{\dfrac{2\tan\theta}{1-\tan^2\theta}+\tan\theta}{1-\dfrac{2\tan\theta}{1-\tan^2\theta}\times\tan\theta}$	M1	2.1
	$= \dfrac{2\tan\theta + \tan\theta\left(1-\tan^2\theta\right)}{1-\tan^2\theta - 2\tan\theta\times\tan\theta}$	M1	1.1b
	$= \dfrac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}$ *	A1*	2.1
		(4)	
(b)	$\tan 3\beta = \dfrac{3\times\sqrt{6}-6\sqrt{6}}{1-3\times 6} = \dfrac{3}{17}\sqrt{6}$	M1 A1	1.1b 2.1
		(2)	
			(6 marks)

Notes:

(a)

B1: States or uses $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$. This may be unsimplified ie. $\tan 2\theta = \dfrac{\tan\theta+\tan\theta}{1-\tan\theta\tan\theta}$

M1: Attempt to use the identity $\tan(A+B)$ with $A=2\theta$ and $B=\theta$ or vice versa with $\tan 2\theta$ being replaced by $\dfrac{\tan\theta+\tan\theta}{1-\tan\theta\tan\theta}$. Condone sign slips only on $\tan 3\theta = \dfrac{\tan 2\theta+\tan\theta}{1-\tan 2\theta\tan\theta}$

M1: Attempts to create a simplified fraction by multiplying both numerator and denominator by $\left(1-\tan^2\theta\right)$ or equivalent

A1*: Shows careful work leading to $\dfrac{3\tan\theta-\tan^3\theta}{1-3\tan^2\theta}$

(b)

M1: Substitutes $\tan\beta=\sqrt{6}$ into the identity for $\tan 3\beta$ in terms of $\tan\beta$

A1: Shows careful work leading to $\tan 3\beta = \dfrac{3}{17}\sqrt{6}$

<table>
<tr><th>Question</th><th colspan="2">Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td>7 (a)</td><td colspan="2">Finds the gradient of the line $\dfrac{2.4-2}{20}=(0.02)$</td><td>M1</td><td>1.1b</td></tr>
<tr><td></td><td colspan="2">States $\log_{10} P = 0.02t+2$</td><td>A1</td><td>2.5</td></tr>
<tr><td></td><td colspan="2"></td><td>(2)</td><td></td></tr>
<tr><td>(b)</td><td colspan="2">Uses the model to deduces that $\log_{10} P_0 = 2$</td><td>M1</td><td>3.4</td></tr>
<tr><td></td><td colspan="2">Initial population is 100</td><td>A1</td><td>1.1b</td></tr>
<tr><td></td><td colspan="2"></td><td>(2)</td><td></td></tr>
<tr><td>(c)</td><td>Uses $\log_{10} P = 0.02t+2$
$\Rightarrow P = 10^2 \times \left(10^{0.02}\right)^t$
$\Rightarrow b = 10^{0.02}$</td><td>Uses $P = ab^t$
$\Rightarrow \log_{10} P = \log_{10} a + t\log_{10} b$
$\Rightarrow \log_{10} b = 0.02$</td><td>M1</td><td>2.1</td></tr>
<tr><td></td><td>$b = 1.047$
$P = 100 \times 1.047^t$</td><td>$b\left(=10^{0.02}\right)=1.047$
$P = 100 \times 1.047^t$</td><td>A1</td><td>1.1b</td></tr>
<tr><td></td><td colspan="2"></td><td>(2)</td><td></td></tr>
<tr><td>(d)</td><td colspan="2">(i) Substitutes $t = 23$ into $\log_{10} P = 0.02t+2$ or
$P = 100 \times 1.047^t$
and proceeds to find a value for P</td><td>M1</td><td>3.4</td></tr>
<tr><td></td><td colspan="2">Achieves 288 or 289 red squirrels and states that this is significantly more than 198 so model is not valid in 2019</td><td>A1</td><td>3.5a</td></tr>
<tr><td></td><td colspan="2">(ii) Gives a suitable reason that explains why there are fewer squirrels
• the wood may only be big enough to sustain a certain population
• a predator/ competitor may have moved into the wood
• weather effects, or disease, may have reduced the numbers after 2016</td><td>B1</td><td>3.5b</td></tr>
<tr><td></td><td colspan="2"></td><td>(3)</td><td></td></tr>
<tr><td colspan="5" align="right">(9 marks)</td></tr>
</table>

Notes:

(a)

M1: Attempts to find the gradient of the line suing the points $(0,2)$ and $(20,2.4)$

Condone use of $y = mx+2$ with the point $(20,2.4)$

A1: States $\log_{10} P = 0.02t+2$ using correct notation

(b)

M1: Uses the model to deduce that the initial population of red squirrels is $\log_{10} P_0 = 2$ or 10^2

A1: States 100. This alone scores both marks

(c)

M1: Uses clear reasoning to proceed to an equation for *b*.

Starting at $\log_{10} P = 0.02t + 2$ it requires correct use of inverse logs and index work to arrive at $b = 10^{0.02}$

Starting at $P = ab^t$ or their $P = 100b^t$ it requires taking $\log_{10}$'s and using correct log work to arrive at

$\log_{10} b = 0.02$

A1: $b = 1.047$ leading to the equation of the model $P = 100 \times 1.047^t$

(d)(i)

M1: Uses t = 23 in the model $\log_{10} P = 0.02t + 2$ or their $P = 100 \times 1.047^t$ and proceeds as far as P =...

A1: Achieves either 288 or 289 red squirrels, states that this is significantly more than the number of squirrels that are present, so model in not valid in 2019

This mark requires a correct number of squirrels, a correct reason and a (minimal) conclusion

(d)(ii)

B1: Gives a suitable reason that explains why there are fewer squirrels than the model would predict

Question	Scheme	Marks	AOs
8 (a)	**(a)** $P=(210,-5)$	B1 B1	1.1b 1.1b
		(2)	
(b)	$5\cos(x-30°)=4\sin x$		
	Uses the compound angle identity and attempts to collect terms $5\cos x\cos 30°+5\sin x\sin 30°=4\sin x$ $5\cos x\cos 30°=\sin x(4-5\sin 30°)$	M1	2.1
	Uses $\frac{\sin x}{\cos x}=\tan x$ $\tan x=\frac{5\cos 30°}{4-5\sin 30°}=\frac{5\sqrt{3}}{3}$ or awrt 2.89	M1 A1	2.1 1.1b
	$x=70.9, 250.9$	A1	1.1b
		(4)	
(c)	Deduces the number of roots = 40	B1ft	2.2a
	Offers a correct explanation. Eg States that there are **10** cycles of $360°$ and $x\to 2x$ will mean that there are 4 roots between 0 and $360°$ (not 2)	B1	2.4
		(2)	
			(8 marks)

Notes:

(a)

B1: For one correct coordinate. Either $P=(...,-5)$ or $P=(210,...)$ but allow $P=(210°,...)$

B1: For $P=(210,-5)$ but allow $P=(210°,-5)$

(b)

M1: Attempts to use the identity $5\cos(x-30°)\equiv 5\cos x\cos 30°+5\sin x\sin 30°$ and attempts to collect terms . If the candidate divides by $\cos x$ first then it is for collecting terms in $\tan x$

M1: Divides by $\cos x$, uses the identity $\tan x=\frac{\sin x}{\cos x}$ leading to $\tan x=...$

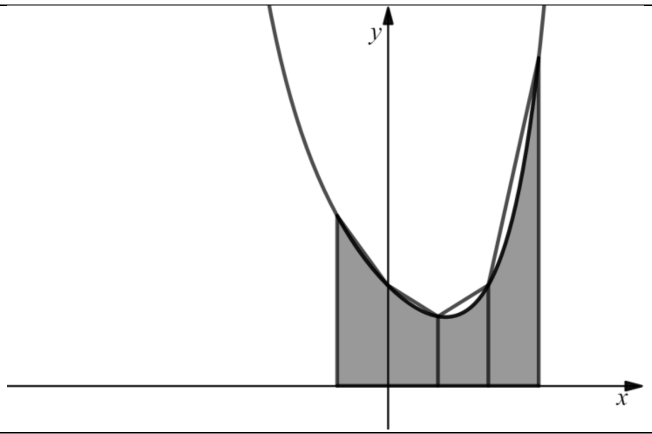
A1: $\tan x=\frac{5\sqrt{3}}{3}$ but allow $\tan x=$ awrt 2.89

A1: Both $x=70.9, 250.9$ (awrt 1dp) and no other answers. Allow $x=70.9°, 250.9°$

(c)

B1ft: Deduces that there will be 40 roots but follow through on $20\times$ the number of roots the candidate has between 0 and $360°$ in (b)

B1: Explains that $3600°$ is **10** cycles of $360°$ **and** $x\to 2x$ will **double** the number roots between 0 and $360°$

Question	Scheme	Marks	AOs
9(a)	Calculates any 2 of the necessary 5 y values $1.689, 1, 0.689, 1, 3.257$ Allow accuracy to 2 dp rounded or truncated	B1	1.1b
	Correct strip width $h = 0.5$	B1	1.1b
	Correct application of the trapezium rule with their values given to at least 2dp rounded or truncated $\frac{0.5}{2}\{1.689+3.257+2\times(1+0.689+1)\}$	M1	2.1
	$= 2.58$	A1	1.1b
		(4)	
(b)	 Draws trapezia on Diagram 1 and states that it is an overestimate as the area of the trapezia are greater than the area of R.	B1	2.4
		(1)	
(c)	$\int_{-0.5}^{1.5}\left(2^{x^2+1}+2x\right)\mathrm{d}x = \int_{-0.5}^{1.5}\left(2\times 2^{x^2}+2x\right)\mathrm{d}x$ $= 2\int_{-0.5}^{1.5}\left(2^{x^2}-x\right)\mathrm{d}x + \int_{-0.5}^{1.5} 4x\,\mathrm{d}x$	M1	3.1a
	$= 2\times 2.58 + \left[2x^2\right]_{-0.5}^{1.5}$	A1ft	1.1b
	= 5.16+ 4 = 9.16 **or** 2 × "(a)" + 4	A1ft	2.1
		(3)	
			(8 marks)

Notes:

(a)

B1: For finding at least two of the five y values that are required to apply the trapezium rule with four strips.

B1: For a strip width of 0.5. This may be implied by sight of $\frac{0.5}{2}\{......\}$ or $0.25\{......\}$

M1: For a full and correct application of the trapezium rule with correct strip width and correct attempt at the y values with an accuracy of at least 2 dp rounded or truncated.

A1: awrt 2.58

(b)

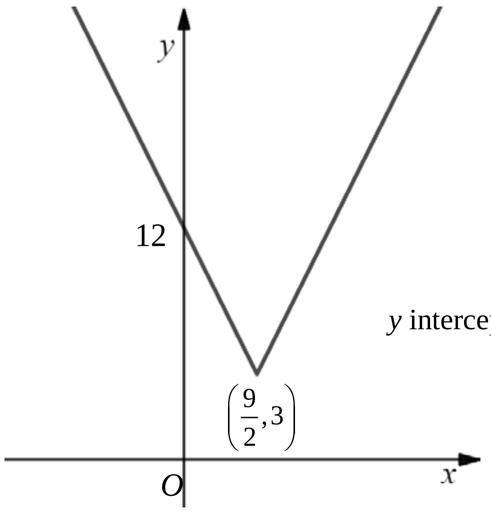
B1: Appropriate diagram and explains that the area of each trapezium is greater than the area of the corresponding strip and so concludes that the answer to (a) is an overestimate.

(c)

M1: For the strategy of using the answer to part (a) in an attempt to find (c). Expect to see $2^{x^2+1} = 2\times 2^{x^2}$

followed by, for example, $\int_{-0.5}^{1.5}\left(2^{x^2+1}+2x\right)\mathrm{d}x=\int_{-0.5}^{1.5}\left(2\times 2^{x^2}+2x\right)\mathrm{d}x=2\times\int_{-0.5}^{1.5}\left(2^{x^2}-x\right)\mathrm{d}x\pm\int_{-0.5}^{1.5}kx\,\mathrm{d}x$
A1ft: $2\times(a)+\left[2x^2\right]_{-0.5}^{1.5}$ following through on their answer to part (a)
A1ft: For careful and accurate work leading to an answer of 9.16 units **or** 2 × their (a) + 4

Question	Scheme	Marks	AOs

Question	Scheme	Marks	AOs
10 (a)	 V Shape	M1	1.1b
	y intercept at 12 or vertex at $\left(\frac{9}{2},3\right)$	B1	1.1b
	Completely correct	A1	1.1b
		(3)	
(b)	Attempts to solve $3x+1=9-2x+3$	M1	1.1b
	$x=\frac{11}{5}$ only	A1	2.1
		(2)	
(c)	Deduces that it will not meet if $k \geqslant -2$	B1	2.2a
	Attempts to find the value of k using their $\left(\frac{9}{2},3\right)$ $3=k\times\frac{9}{2}+1\Rightarrow k=$	M1	3.1a
	$-2 \leqslant k<\frac{4}{9}$	A1	3.2a
		(3)	
		(8 marks)	

Notes:

(a)

See scheme

(b)

M1: Attempts to solve $3x+1=9-2x+3$. (There may also be an attempt to solve $3x+1=2x-9+3$ which can be ignored for this mark.)

A1: $x=\frac{11}{5}$ **only.** Extra solutions (unless deleted) will be penalised.

(c)

B1: Deduces that they cannot meet if the value of k is greater than or equal to -2 This may be awarded within an inequality. Condone $k>-2$ for this mark

M1: Attempts to find the gradient of the line with intercept 1 that passes through their minimum point.

A1: $-2 \leqslant k<\frac{4}{9}$

Question	Scheme	Marks	AOs
11 (a)	Attempts $\mathrm{f}\left(\frac{5}{3}\right)=\dfrac{2+3\times\frac{5}{3}}{\frac{5}{3}}=$ or attempts to solve $\dfrac{2}{a-3}=\dfrac{5}{3}$	M1	3.1a
	$\dfrac{21}{5}$ oe	A1	1.1b
		(2)	
(b)	$\dfrac{2+3x}{x}<\dfrac{3}{2}\Rightarrow x<-\dfrac{4}{3}$	M1	3.1a
	There are no values as f is only defined for $x>0$	A1	3.2a
		(2)	
(c)	Attempts $\mathrm{f}'(x)=\dfrac{x\times3-(2+3x)}{x^2}$	M1	1.1b
	$\mathrm{f}'(x)=\dfrac{-2}{x^2}<0$ so f is decreasing for all values of x	A1	2.4
		(2)	
			(6 marks)

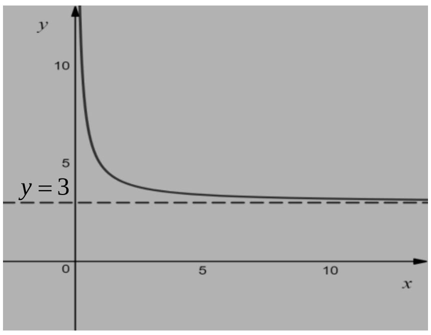
Notes:

(a)

M1: For a valid method. Accept an attempt at $\mathrm{f}\left(\frac{5}{3}\right)$ or else an attempt to find $\mathrm{f}^{-1}(a)$ by change of subject followed by an attempt at solving $\mathrm{f}^{-1}(a)=\dfrac{5}{3}$

A1: $\dfrac{21}{5}$

(b)



M1: Attempts to solve $\mathrm{f}(x)<\dfrac{3}{2}$ or sketches graph and shows the asymptote. Alt writes $\mathrm{f}(x)=3+\dfrac{2}{x}$ and states f >3

A1: States that there are no values and gives a valid reason. When using a graphical solution "stating $\mathrm{f}(x)>3$ hence there are no values for $\mathrm{f}(x)<\dfrac{3}{2}$" is sufficient.

(c)

M1: Award for an attempt to find $\mathrm{f}'(x)$ using the quotient rule or via an explanation involving the changed function $\mathrm{f}(x)=3+\dfrac{2}{x}$ (As x increases $\dfrac{2}{x}$ decreases...)

A1: Requires a correct calculation and explanation

<table>
<tr><th>Question</th><th colspan="2">Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td rowspan="5">12</td><td>$\frac{4x+9}{x+3}=4-\frac{3}{x+3}$</td><td>Let $u=x+3$
$\int \frac{4x+9}{x+3}\,\mathrm{d}x=\int \frac{4(u-3)+9}{u}\,\mathrm{d}u$</td><td>M1</td><td>1.1b</td></tr>
<tr><td>$\int 4-\frac{3}{x+3}\,\mathrm{d}x$
$=4x-3\ln(x+3)$</td><td>$=\int 4-\frac{3}{u}\,\mathrm{d}u$
$4u-3\ln u$</td><td>dM1
A1</td><td>2.1
1.1b</td></tr>
<tr><td>Uses limits 5 and 1 and uses correct ln work
$=20-3\ln 8-4+3\ln 4=\ldots$</td><td>Uses limits 8 and 4 and uses correct ln work
$=32-3\ln 8-16+3\ln 4=\ldots$</td><td>ddM1</td><td>1.1b</td></tr>
<tr><td colspan="2">$16-3\ln 2$ or $16+3\ln\left(\frac{1}{2}\right)$ oe</td><td>A1</td><td>2.1</td></tr>
<tr><td colspan="2"></td><td>(5)</td><td></td></tr>
<tr><td colspan="5" align="right">(5 marks)</td></tr>
</table>

Notes

M1: Attempts to use an appropriate method to integrate the function.

If division is used look for $4\pm\frac{k}{x+3}$

If substitution is used look for $u=x+3$ and a full attempt to get the integral in x into an integral in u

dM1: Dependent upon the previous t M mark. It is for the full method of integrating the function.

For division look for $4x\pm k\ln(x+3)$

For substitution look for $4u\pm k\ln u$

A1: Fully correct integration

M1: For the key steps of using the correct limits for their function with correct ln work to get a simplified expression

A1: $16-3\ln 2$ or $16+3\ln\left(\frac{1}{2}\right)$ oe such as $16-\ln 8$

Question	Scheme	Marks	AOs
13 (a)	Uses $V=\frac{1}{3}\pi r^2 h$ with $\frac{r}{h}=\frac{2.5}{4}$ to establish $V=\mathrm{f}(h^3)$ and differentiates	M1	2.1
	$\frac{\mathrm{d}V}{\mathrm{d}h}=\frac{75}{192}\pi h^2$	A1	1.1b
	States or uses $\frac{\mathrm{d}V}{\mathrm{d}t}=-\frac{\pi}{512}\sqrt{h}$	B1	1.1b
	Uses $\frac{\mathrm{d}V}{\mathrm{d}t}=\frac{\mathrm{d}V}{\mathrm{d}h}\times\frac{\mathrm{d}h}{\mathrm{d}t}$ with their $\frac{\mathrm{d}V}{\mathrm{d}t}$ and their $\frac{\mathrm{d}V}{\mathrm{d}h}$	M1	3.1b
	$--\frac{\pi}{512}\sqrt{h}=\frac{75}{192}\pi h^2\times\frac{\mathrm{d}h}{\mathrm{d}t}\Rightarrow h^{\frac{3}{2}}\frac{\mathrm{d}h}{\mathrm{d}t}=-\frac{1}{200}$ *	A1*	2.1
		(5)	
(b)	$\int h^{\frac{3}{2}}\mathrm{d}h=-\int\frac{1}{200}\mathrm{d}t\Rightarrow\frac{2}{5}h^{\frac{5}{2}}=-\frac{1}{200}t+c$	M1	1.1b
	Substitutes $t=0, h=4\Rightarrow c=\left(\frac{64}{5}\right)$	dM1	3.4
	$\frac{2}{5}h^{\frac{5}{2}}=-\frac{1}{200}t+\frac{64}{5}$ oe	A1	3.3
		(3)	
(c)	Substitutes $h=0\Rightarrow 0=-\frac{1}{200}t+\frac{64}{5}\Rightarrow t=\ldots$	M1	3.4
	$t=2560$ seconds = 42 minutes 40 seconds	A1	3.2a
	States that the "real" time and the "predicted" times are very close so model seems suitable	A1	3.5a
		(3)	
			(11 marks)

Notes:

(a)

M1: Uses $V=\frac{1}{3}\pi r^2 h$ with $\frac{r}{h}=\frac{2.5}{4}$ or equivalent to establish V as a function of h^3 which is then differentiated to an expression in h^2

A1: $\frac{\mathrm{d}V}{\mathrm{d}h}=\frac{75}{192}\pi h^2$

B1: Uses the information given in the question to states or uses $\frac{\mathrm{d}V}{\mathrm{d}t}=-\frac{\pi}{512}\sqrt{h}$

M1: Uses $\frac{\mathrm{d}V}{\mathrm{d}t}=\frac{\mathrm{d}V}{\mathrm{d}h}\times\frac{\mathrm{d}h}{\mathrm{d}t}$ with their $\frac{\mathrm{d}V}{\mathrm{d}t}$ and their $\frac{\mathrm{d}V}{\mathrm{d}h}$ to form an equation linking $\frac{\mathrm{d}h}{\mathrm{d}t}$ and h

A1*: Proceeds correctly to the given equation $h^{\frac{3}{2}}\frac{\mathrm{d}h}{\mathrm{d}t}=-\frac{1}{200}$

(b)

M1: Integrates both sides to $ah^{\frac{5}{2}} = bt + c$. Condone the omission of + c for this mark
dM1: Uses the model to find c
A1: Finds the equation of the model Eg. $\frac{2}{5}h^{\frac{5}{2}} = -\frac{1}{200}t + \frac{64}{5}$ or equivalent such as $h^{\frac{5}{2}} = -\frac{1}{80}t + 32$
(c)
M1: Uses the model with $h = 0$ and proceeds to find t
A1: Achieves $t = 2560$ seconds and converts this or the 43 minutes as required in order to test the model
A1: States that the "real" time and the "predicted" times are very close so the model seems suitable

Question	Scheme	Marks	AOs
14(a)	Uses $y = 3\sin 2t = 6\sin t\cos t$ and attempts to square	M1	2.1
	$y^2 = 9x^2\cos^2 t$	A1	1.1b
	Uses $\cos^2 t = 1 - \sin^2 t$ with $\sin t = \frac{x}{2}$ $y^2 = 9x^2\left(1 - \frac{x^2}{4}\right)$	M1	2.1
	$y^2 = \frac{9}{4}x^2\left(4 - x^2\right)$	A1	1.1b
		(4)	
(b)	Deduces that the radius of the circle is given by $r^2 = x^2 + y^2$	M1	3.1a
	$r^2 = x^2 + \frac{9}{4}x^2\left(4 - x^2\right)$	A1	1.1b
	Circle touches curve when r is a maximum so differentiate $r^2 = 10x^2 - \frac{9}{4}x^4 \Rightarrow 2r\frac{\mathrm{d}r}{\mathrm{d}x} = 20x - 9x^3$ and set $\frac{\mathrm{d}r}{\mathrm{d}x} = 0 \Rightarrow x = \sqrt{\frac{20}{9}}$	M1	3.1a
	Finds $r^2 = 10x^2 - \frac{9}{4}x^4$ with their $x = \sqrt{\frac{20}{9}}$	dM1	2.1
	$r = \frac{10}{3}$	A1	1.1b
		(5)	
			(9 marks)

(a)

M1: Uses $\sin 2t = 2\sin t\cos t$ and attempts to square $y = 3\sin 2t$

A1: $y^2 = 9x^2\cos^2 t$

M1: Uses $\cos^2 t = 1 - \sin^2 t$ with $\sin t = \frac{x}{2}$ in an attempt to get an equation in just x and y

A1: $y^2 = \frac{9}{4}x^2\left(4 - x^2\right)$

Note: It is possible to use the given equation $y^2 = kx^2\left(4 - x^2\right)$. The first M1 is scored for substituting in both x and y and using $\sin 2t = 2\sin t\cos t$. The second M1 is for using $4 - 4\sin^2 t = 4\cos^2 t$. For the A1 to be awarded there must be some minimal statement such as ✓ it can be expressed in this form

(b)

M1: For deducing that the radius of the circle can be found from $x^2 + y^2$

A1: A correct statement for r^2 in terms of one variable.

Look for $r^2 = x^2 + \frac{9}{4}x^2\left(4 - x^2\right)$ or $r^2 = 4\sin^2 t + 9\sin^2 2t$

M1: For a full method of finding a value of *x* or *t* where the circle touches the curve.

For this to be scored expect to see an attempt at implicit differentiation (or equivalent) followed by an attempt to find where $\frac{\mathrm{d}r}{\mathrm{d}x} = 0$.

Also scored for an attempt at completing the square $r^2 = 10x^2 - \frac{9}{4}x^4 = -\frac{9}{4}\left(x^2 - \frac{20}{9}\right)^2 + \frac{100}{9}$

M1: For showing all steps required to find r or r^2

A1: $r = \frac{10}{3}$

(b)	Deduces that the radius of the circle is given by $r^2 = x^2 + y^2$	M1	2.2a
	$r^2 = 4\sin^2 t + 9\sin^2 2t$	A1	1.1b
	Circle touches curve when r is a maximum so differentiate $\Rightarrow 2r\frac{\mathrm{d}r}{\mathrm{d}t} = 8\sin t\cos t + 36\sin 2t\cos 2t$ And set $\frac{\mathrm{d}r}{\mathrm{d}t} = 0 \Rightarrow 0 = 4\times \cancel{2\sin t\cos t} + 36\cancel{\sin 2t}\cos 2t$	M1	3.1a
	Finds $r^2 = 4\sin^2 t + 9\sin^2 2t$ with their $\cos 2t = -\frac{1}{9}$	dM1	2.1
	$\cos 2t = -\frac{1}{9} \Rightarrow \sin^2 t = \frac{5}{9}$ and $r^2 = 4\times\frac{5}{9} + 9\left(1 - \frac{1}{81}\right) = \frac{100}{9}$	A1	1.1b
		(5)	

Alt via simultaneous equations:

M1: Solves $r^2 = x^2 + y^2$ and $y^2 = \frac{9}{4}x^2\left(4 - x^2\right)$ to get an equation in either *x* or *y*.

A1: Either $9x^4 - 40x^2 + 4r^2 = 0$ or $9y^4 + \left(40 - 18r^2\right)y^2 + 9r^4 - 36r^2 = 0$

M1: Attempts $b^2 - 4ac = 0$ for their quartic equation of a form $ax^4 + bx^2 + c = 0$ where either *a*, *b* or *c* are dependent upon *r*

dM1: Uses $b^2 - 4ac = 0$ for their quartic equation of a form $ax^4 + bx^2 + c = 0$ where either *a*, *b* or *c* are dependent upon r to find a value for *r*

A1: $r = \frac{10}{3}$

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