****

**1.** Curve *C* has equation

*y* = *x*3 – 7*x*2 + 5*x* + 4

(*a*)Find 

**(2)**

The point *P* (2, –6) lies on *C*

(*b*)Find the equation of the tangent to *C* at *P*

Give your answer in the form *y* = *mx* + *c* where *m* and *c* are integers to be found.

**(3)**

**(Total for Question 1 is 5 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**2.** f(*x*) = 3*x*3 *–* 7*x*2 + 7*x* – 10

(*a*)Use the factor theorem to show that (*x* – 2) is a factor of f(*x*)

**(2)**

(*b*)Find the values of the constants *a*, *b* and *c* such that

f(*x*) ≡ (*x* – 2)(*ax*2 + *bx* + *c*)

**(3)**

(*c*)Using your answer to part (*b*)show that the equation f(*x*) = 0 has only one real root.

**(2)**

**(Total for Question 2 is 7 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**3.**



Figure 1 shows a sketch of part of the curve with equation *y* = f(*x*)

The table below shows corresponding values of *x* and *y* for this curve between *x* = 0.5

and *x* = 0.9

The values of *y* are given to 4 significant figures.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| *x* | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 |
| *y* | 1.632 | 1.711 | 1.786 | 1.859 | 1.930 |

(*a*)Use the trapezium rule, with all the values of *y* in the table, to find an estimate for



Give your answer to 3 significant figures.

**(3)**

(*b*)Using your answer to part (*a*), deduce an estimate for



**(3)**

**(Total for Question 3 is 6 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**4. In this question you must show all stages of your working.**

 **Solutions relying entirely on calculator technology are not acceptable.**

(*a*)Express as an integral

****

**(1)**

(*b*)Using your answer to part (*a*)show that

****

**(3)**

**(Total for Question 4 is 4 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**5.** The functions f and g are defined by

  *x* ∈ ℝ *x* ≠ 

 g(*x*) = 2 + 3*x* – *x*2 *x* ∈ ℝ

where *k* is a non‑zero constant.

(*a*)Find in terms of *k*

(i) fg(4)

(ii) the range of f

(iii) f –1

**(6)**

Given that



(*b*)find the exact value of *k*

**(2)**

**(Total for Question 5 is 8 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**6.** 

(*a*)Show that the first 4 terms in the binomial expansion of f(*x*), in ascending powers

of *x*, are

*A* + *Bx* + *Cx*2 + 

where *A*, *B* and *C* are constants to be found. Give each constant in simplest form.

**(4)**

Given that this expansion is valid for | *x*| < *k*

(*b*)state the largest value of *k*.

**(1)**

By substituting *x* =  into f(*x*) and into the answer for part (*a*),

(*c*)find an approximation for 

Give your answer in the form  where *a* and *b* are integers to be found.

**(2)**

**(Total for Question 6 is 7 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**7.** Curve *C* has equation

*y* = (*x*2– 5*x* + 8) *x* ∈ ℝ

(*a*)Show that



**(3)**

Given that

* *C* has only one stationary point
* the stationary point has *x* coordinate *α*
*  at *x* = 0.3
*  at *x* = 0.4

(*b*)explain why 0.3 < *α* < 0.4

**(1)**

(*c*)Show that *α* is a solution of the equation



**(3)**

(*d*)Using the iteration formula

 with *x*1 = 0.3

find

(i) the value of *x*3 to 4 decimal places,

(ii) the value of *α* to 4 decimal places.

**(3)**

**(Total for Question 7 is 10 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**8. In this question you must show all stages of your working.**

 **Solutions relying entirely on calculator technology are not acceptable.**

(*a*)Show that

  *n* ∈ ℤ

where *k* is a constant to be found.

**(3)**

(*b*)Hence solve, for 



Give your answers to 3 significant figures where appropriate.

**(4)**

**(Total for Question 8 is 7 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**9.** The vertical speed, *v* m s–1, of a skydiver, *t* seconds after their parachute opened, is

modelled by the equation

*v* = *A* + *B*e–0.5*t*

where *A* and *B* are constants.

Given that the vertical speed of the skydiver was

* 56 m s–1 at the instant the parachute opened
* 10 m s–1 exactly 5 seconds after the parachute opened

(*a*)find a complete equation for the model.

Give the values of *A* and *B* to 3 significant figures.

**(4)**

Given also that the skydiver eventually descended safely to the ground at a constant

vertical speed of 6 m s–1

(*b*)evaluate the model.

**(2)**

**(Total for Question 9 is 6 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**10.** (*a*)Sketch the graph with equation

*y* = |3*x* – 2*a*|

where *a* is a positive constant.

State the coordinates of each point where the graph cuts or meets the

coordinate axes.

**(2)**

(*b*)Solve, in terms of *a*, the inequality

| 3*x* – 2*a*| ≤ *x + a*

**(4)**

Given that |3*x* – 2*a*| ≤ *x + a*

(*c*)find, in terms of *a*, the range of possible values of g(*x*), where



**(3)**

**(Total for Question 10 is 9 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**11.** The mean yearly concentration, *C* parts per million (ppm), of carbon dioxide in the

Earth’s atmosphere was first measured in 1960.

The equation

*C* = *abt* where *a* and *b* are constants

models the mean yearly concentration of carbon dioxide *t* years after 1960.

Given that the mean yearly concentration of carbon dioxide was

* 339 ppm in 1980
* 414 ppm in 2020

(*a*)(i) find the value of *b* to 3 decimal places,

(ii) find the value of *a* to the nearest integer.

**(4)**

(*b*)With reference to the model,

(i) interpret the value of *a*,

(ii) interpret the value of *b*.

**(2)**

Using the model,

(*c*)find the year when the mean yearly concentration of carbon dioxide is predicted to

reach 450 ppm.

(*Solutions based entirely on calculator technology are not acceptable*.)

**(4)**

**(Total for Question 11 is 10 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**12. In this question you must show all stages of your working.**

 **Solutions relying entirely on calculator technology are not acceptable.**

(*a*)Show that



where *k* is a constant and where *c* is an arbitrary constant.

**(3)**

A theme park ride lasts for 70 seconds.

The height above ground, *H* metres, of a passenger on the theme park ride is modelled

by the differential equation

 0 ≤ *t* ≤ 70

where *t* seconds is the time from the start of the ride.

Given that the passenger is 5 m above ground at the start of the ride

(*b*)find the height above ground of the passenger 52 seconds after the start of the ride.

**(6)**

**(Total for Question 12 is 9 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**13.** Given that

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show that



**(5)**

**(Total for Question 13 is 5 marks)**

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**14.** (i) Prove that the sum of the squares of 2 consecutive odd integers is always 2 more

than a multiple of 8

**(3)**

(ii) Use proof by contradiction to show that log25 is irrational.

**(4)**

**(Total for Question 14 is 7 marks)**

**(TOTAL FOR PAPER IS 100 MARKS)**