Pearson Edexcel

# Mark Scheme

# Mock Set 3

Pearson Edexcel GCE Mathematics

Pure 1 Paper 9MA0/01

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**General Marking Guidance**

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

**EDEXCEL GCE MATHEMATICS**
**General Instructions for Marking**

1. The total number of marks for the paper is 100.

2. The Edexcel Mathematics mark schemes use the following types of marks:
   - **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
   - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - **B** marks are unconditional accuracy marks (independent of M marks)
   - Marks should not be subdivided.

3. Abbreviations
   These are some of the traditional marking abbreviations that will appear in the mark schemes.

   - bod – benefit of doubt
   - ft – follow through
   - the symbol $\sqrt{}$ will be used for correct ft
   - cao – correct answer only
   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - oe – or equivalent (and appropriate)
   - dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - ∗ The answer is printed on the paper
   - ⎣ The second mark is dependent on gaining the first mark

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses and indicates which response they wish to submit, examiners should mark this response.
   If there are several attempts at a question which have not been crossed out, examiners should mark the final answer which is the answer that is the most complete.

6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

## General Principles for Further Pure Mathematics Marking

(*But note that specific mark schemes may sometimes override these general principles*)

**<u>Method mark for solving 3 term quadratic:</u>**

**1. Factorisation**

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \ldots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \ldots$

**2. Formula**

Attempt to use the correct formula (with values for *a*, *b* and *c*)

**3. Completing the square**

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0, \quad q \neq 0$, leading to $x = \ldots$

**<u>Method marks for differentiation and integration:</u>**

**1. Differentiation**

Power of at least one term decreased by 1. $(x^n \to x^{n-1})$

**2. Integration**

Power of at least one term increased by 1. $(x^n \to x^{n+1})$

**<u>Use of a formula</u>**

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

<u>Method mark</u> for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is <u>not</u> quoted, the method mark can be gained by implication from <u>correct</u> working with values but may be lost if there is any mistake in the working.

**<u>Exact answers</u>**

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **1(a)** | $y = x^3 - 7x^2 + 5x + 4 \Rightarrow \frac{dy}{dx} = 3x^2 - 14x + 5$ | M1<br>A1 | 1.1b<br>1.1b |
| | | **(2)** | |
| **(b)** | $x = 2 \Rightarrow \frac{dy}{dx} = 3(2)^2 - 14(2) + 5 = -11$ | M1 | 1.1b |
| | $y + 6 = -11(x - 2)$ | M1 | 1.1b |
| | $y = -11x + 16$ | A1 | 1.1b |
| | | **(3)** | |
| | | | **(5 marks)** |

**Notes**

(a)

M1: $x^n \to x^{n-1}$ at least once

A1: Correct derivative

(b)

M1: Attempts the gradient at *P*

M1: Complete method for the equation of the tangent using their gradient at *P* and (2, −6)

A1: Correct equation

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **2(a)** | $\mathrm{f}(2)=3(2)^3-7(2)^2+7(2)-10=...$ | M1 | 1.1b |
| | $\mathrm{f}(2)=38-38=0 \Rightarrow (x-2)$ is a factor of $\mathrm{f}(x)$ * | A1* | 2.1 |
| | | **(2)** | |
| **(b)** | $a=3$ or $c=5$ | B1 | 2.2a |
| | $\mathrm{f}(x)=(x-2)(...x^2+...x+...)$ | M1 | 1.1b |
| | $a=3,\ b=-1,\ c=5$ | A1 | 1.1b |
| | | **(3)** | |
| **(c)** | $3x^2-x+5=0 \Rightarrow b^2-4ac=(-1)^2-4(3)(5)=...$<br>or e.g.<br>$3x^2-x+5=0 \Rightarrow 3\left(x^2-\frac{1}{3}x+\frac{5}{3}\right)=0 \Rightarrow \left(x-\frac{1}{6}\right)^2-\frac{1}{36}+\frac{5}{3} \Rightarrow \left(x-\frac{1}{6}\right)^2=...$<br>or e.g.<br>$\frac{\mathrm{d}y}{\mathrm{d}x}=6x-1=0 \Rightarrow x=..., \Rightarrow y=...$ | M1 | 3.1a |
| | $(-1)^2-4(3)(5)=-59 \Rightarrow b^2-4ac<0$<br>or e.g.<br>$\left(x-\frac{1}{6}\right)^2=-\frac{59}{36}$ and square numbers cannot be negative<br>or e.g.<br>$\frac{\mathrm{d}y}{\mathrm{d}x}=0 \Rightarrow y=\frac{59}{12}$ so the minimum is above the $x$-axis<br>So the quadratic has no real roots and so $\mathrm{f}(x)=0$ has only 1 real root | A1 | 2.4 |
| | | **(2)** | |
| | | **(7 marks)** | |

**Notes**

(a)
M1: Attempts $\mathrm{f}(2)$
A1*: Clearly shows $\mathrm{f}(2)=0$ and makes a suitable conclusion
(b)
B1: Deduces the correct value of $a$ or $c$
M1: Complete method to obtain values for $a$, $b$ and $c$. May use inspection or expand to give $ax^3+(b-2a)x^2+(c-2b)x-2c$ and compare coefficients.
A1: All correct stated or embedded
(c)
M1: Starts the process of showing that their 3-term quadratic has no real roots. E.g. considers discriminant or attempts to solve by completing the square or differentiates to find turning point
A1: Fully correct work with appropriate conclusion for their chosen method

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **3(a)** | $h = 0.1$ | B1 | 1.1a |
| | $A \approx \frac{0.1}{2}\{1.632 + 1.930 + 2(1.711 + 1.786 + 1.859)\}$ | M1 | 1.1b |
| | $= 0.714$ | A1 | 1.1b |
| | | **(3)** | |
| **(b)** | $\int_{0.5}^{0.9} (3\mathrm{f}(x) + 2)\ \mathrm{d}x = 3\times\text{"}0.714\text{"} + \ldots$ | M1 | 1.1b |
| | $\int_{0.5}^{0.9} (3\mathrm{f}(x) + 2)\ \mathrm{d}x = \ldots + 2\times 0.4$ | M1 | 3.1a |
| | $\int_{0.5}^{0.9} (3\mathrm{f}(x) + 2)\ \mathrm{d}x = 3\times\text{"}0.714\text{"} + 0.8 = 2.942$ | A1ft | 2.2a |
| | | **(3)** | |
| | | | **(6 marks)** |

**Notes**

(a)

B1: States or uses $h = 0.1$

M1: Correct attempt at the trapezium rule. Must be an attempt at the correct structure e.g. $\frac{h}{2}\{y_{0.5} + y_{0.9} + 2(y_{0.6} + y_{0.7} + y_{0.8})\}$ with brackets as shown unless they are implied by subsequent work

A1: For awrt 0.714

(b)

M1: For multiplying their answer to part (a) by 3

M1: For a correct strategy for the "+ 2" part of the integral. May see e.g. $2\times 0.4$ or $2\times(0.9 - 0.5)$ or $\int_{0.5}^{0.9} 2\ \mathrm{d}x = [2x]_{0.5}^{0.9} = 2\times 0.9 - 2\times 0.5$

A1ft: For awrt 2.94 or follow through $3\times$their answer to part (a) + 0.8

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **4(a)** | $\int_4^{12} (1+2x)^{\frac{1}{2}}\, dx$ | B1 | 1.2 |
| | | **(1)** | |
| **(b)** | $\int (1+2x)^{\frac{1}{2}}\, dx = k(1+2x)^{\frac{3}{2}}\,(+c)$ | M1 | 1.1b |
| | $\int (1+2x)^{3}\, dx = \frac{1}{3}(1+2x)^{\frac{3}{2}}\,(+c)$ | A1 | 1.1b |
| | $\int_4^{12} (1+2x)^{\frac{1}{2}}\, dx = \left[\frac{1}{3}(1+2x)^{\frac{3}{2}}\right]_4^{12} = \frac{1}{3}(125-27) = \frac{98}{3}$ | A1 | 1.1b |
| | | **(3)** | |
| | | | **(4 marks)** |

**Notes**

(a)

B1: States $\int_4^{12} (1+2x)^{\frac{1}{2}}\, dx$

(b)

M1: Integrates $(1+2x)^{\frac{1}{2}}$ to give $k(1+2x)^{\frac{3}{2}}\,(+c)$

A1: $\int (1+2x)^{\frac{1}{2}}\, dx = \frac{1}{3}(1+2x)^{\frac{3}{2}}\,(+c)$

A1: Fully correct proof with evidence of correct use of both limits

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| 5(a)(i) | $\text{fg}(4)=\text{f}\left(2+3(4)-4^2\right)=\text{f}(-2)=\ldots$ | M1 | 1.1b |
| | $=\frac{2k}{5}$ | A1 | 1.1b |
| (ii) | $y\in\mathbb{R},\quad y\neq\frac{k}{2}$ | B1 | 2.2a |
| (iii) | $y=\frac{kx}{2x-1}\Rightarrow 2xy-y=kx\Rightarrow x(2y-k)=y$ | M1 | 2.1 |
| | $\text{f}^{-1}(x)=\frac{x}{2x-k}$ | A1 | 2.5 |
| | $x\neq\frac{k}{2}$ | B1ft | 2.5 |
| | | **(6)** | |
| (b) | $\text{f}^{-1}(2)=\frac{11}{3\text{g}(2)}\Rightarrow\frac{2}{4-k}=\frac{11}{3(4)}\Rightarrow k=\ldots$ | M1 | 3.1a |
| | $24=44-11k\Rightarrow k=\frac{20}{11}$ | A1 | 1.1b |
| | | **(2)** | |
| | (b) Alternative: | | |
| | $\text{f}^{-1}(2)=\frac{11}{3\text{g}(2)}\Rightarrow\text{f}\left(\frac{11}{3\text{g}(2)}\right)=2\Rightarrow\text{f}\left(\frac{11}{12}\right)\Rightarrow k=\ldots$ | M1 | 3.1a |
| | $\frac{11}{10}k=2\Rightarrow k=\frac{20}{11}$ | A1 | 1.1b |
| | | | **(8 marks)** |

**Notes**

(a)(i)
M1: Full method to find g(4) and substitute the result into f. Also allow for an attempt to substitute $x = 4$ into an attempt at fg($x$).
A1: Correct expression
(ii)
B1: Correct range (Allow $x\in\mathbb{R},\ x\neq\frac{k}{2}$)
(iii)
M1: Correct attempt to cross multiply followed by an attempt to factorise out $x$.
A1: Correct expression using the correct notation. Allow $\text{f}^{-1} = \ldots$ or $\text{f}^{-1}: x \rightarrow \ldots$ but not $y = \ldots$
B1ft: The correct domain or follow through their answer to (ii).
(b)
M1: A complete strategy to find $k$.
A1: Deduces the correct exact value

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| 6(a) | $A=5$ | B1 | 2.2a |
| | $\left(1-\frac{3}{4}x\right)^{-\frac{1}{2}} \approx$ $1+\left(-\frac{1}{2}\right)\left(-\frac{3}{4}x\right)+\frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2!}\left(-\frac{3}{4}x\right)^2+\frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)}{3!}\left(-\frac{3}{4}x\right)^3$ | M1<br>A1 | 1.1b<br>1.1b |
| | $\frac{10}{\sqrt{4-3x}} \approx 5+\frac{15}{8}x+\frac{135}{128}x^2+\frac{675}{1024}x^3$ | A1 | 1.1b |
| | | **(4)** | |
| **(b)** | $k=\frac{4}{3}$ | B1 | 2.2a |
| | | **(1)** | |
| **(c)** | $x=\frac{1}{3} \Rightarrow 5+\frac{15}{8}\left(\frac{1}{3}\right)+\frac{135}{128}\left(\frac{1}{3}\right)^2+\frac{675}{1024}\left(\frac{1}{3}\right)^3=\frac{5905}{1024}$<br>$x=\frac{1}{3} \Rightarrow \frac{10}{\sqrt{4-3x}}=\frac{10}{\sqrt{3}} \Rightarrow \sqrt{3} \approx 10 \div \frac{5905}{1024}$ or $\sqrt{3} \approx \frac{3}{10} \times \frac{5905}{1024}=\ldots$ | M1 | 1.1b |
| | $\Rightarrow \sqrt{3} \approx \frac{2048}{1181}$ or $\frac{3543}{2048}$ | A1 | 2.2a |
| | | **(2)** | |
| | | **(7 marks)** | |

**Notes**

(a)

B1: For deducing that $A = 5$. This may be seen as part of their final answer or as e.g.

$$\frac{10}{\sqrt{4-3x}}=\frac{10}{2\sqrt{1-\ldots}} \text{ or } \frac{10}{\sqrt{4-3x}}=10\times\frac{1}{2}(1-\ldots)$$

M1: Uses a correct binomial expansion of their $(1\pm\ldots x)^n$

A1: Correct unsimplified expansion

A1: All correct

Note direct expansion gives:

$$10(4-3x)^{-\frac{1}{2}} \approx 10\left(4^{-\frac{1}{2}}+\left(-\frac{1}{2}\right)\left(4^{-\frac{3}{2}}\right)(-3x)+\left(\frac{-\frac{1}{2}\times-\frac{3}{2}}{2}\right)\left(4^{-\frac{5}{2}}\right)(-3x)^2++\left(\frac{-\frac{1}{2}\times-\frac{3}{2}\times-\frac{5}{2}}{6}\right)\left(4^{-\frac{7}{2}}\right)(-3x)^3\right)$$

Score B1 for "5", M1 for correct structure of the expansion, A1 for correct unsimplified terms and A1 as above

(b)

B1: Deduces the correct value

(c)

M1: Fully correct strategy: Substitutes $x=\frac{1}{3}$ into their expansion and divides into 10 or multiplies by $\frac{3}{10}$

A1: Deduces either value (oe)

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| 7(a) | $\frac{dy}{dx}=(2x-5)e^{x^2}+2x(x^2-5x+8)e^{x^2}$ | M1<br>A1 | 1.1b<br>1.1b |
| | $=(2x^3-10x^2+18x-5)e^{x^2}$ * | A1* | 2.1 |
| | | **(3)** | |
| **(b)** | Sign change, function is continuous therefore 0.3 < α < 0.4 | B1 | 2.4 |
| | | **(1)** | |
| **(c)** | $\frac{dy}{dx}=0 \Rightarrow (2x^3-10x^2+18x-5)e^{x^2}=0$<br>$\Rightarrow 2x^3-10x^2+18x-5=0$ | B1 | 2.2a |
| | $\Rightarrow 2x^3+18x=10x^2+5 \Rightarrow 2x(x^2+9)=5(2x^2+1) \Rightarrow x=...$ | M1 | 2.1 |
| | $x=\frac{5(2x^2+1)}{2(x^2+9)}$ * | A1* | 1.1b |
| | | **(3)** | |
| **(d)(i)** | $x_1=0.3 \Rightarrow x_2=\frac{5(2(0.3)^2+1)}{2((0.3)^2+9)}=...$ | M1 | 1.1b |
| | $x_3=0.3324$ | A1 | 1.1b |
| **(d)(ii)** | $\alpha=0.3364$ | A1 | 2.2a |
| | | **(3)** | |
| | | | **(10 marks)** |

**Notes**

(a)
M1: Correct application of the product rule.
A1: Correct derivative in any form.
A1*: Correct proof with no errors.
(b)
B1: Correct explanation.
(c)
B1: Deduces $2x^3-10x^2+18x-5=0$
M1: Collects appropriate terms to each side and makes *x* the subject.
A1*: Correct proof.
(d)(i)
M1: Substitutes $x$ = 0.3 into the given iterative formula. May be implied by $x_2=\frac{295}{909}$ or $x_2=0.324...$
A1: $x_3=$ awrt 0.3324
(d)(ii)
A1: Correct value as shown.

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| 8(a) | $\frac{1-\cos 2\theta}{\sin^2 2\theta}=\frac{1-\left(1-2\sin^2\theta\right)}{\ldots}$ or $\frac{1-\cos 2\theta}{\sin^2 2\theta}=\frac{\ldots}{\left(2\sin\theta\cos\theta\right)^2}$ | M1 | 1.1b |
| | $\frac{1-\cos 2\theta}{\sin^2 2\theta}=\frac{1-\left(1-2\sin^2\theta\right)}{\left(2\sin\theta\cos\theta\right)^2}=\frac{2\sin^2\theta}{4\sin^2\theta\cos^2\theta}=\ldots$ | M1 | 2.1 |
| | $=\frac{1}{2}\sec^2\theta$ | A1 | 1.1b |
| | | **(3)** | |
| (b) | $\frac{1-\cos 2x}{\sin^2 2x}=\left(1+2\tan x\right)^2 \Rightarrow \frac{1}{2}\sec^2 x=1+4\tan x+4\tan^2 x$ <br> $\Rightarrow 1+\tan^2 x=2+8\tan x+8\tan^2 x \Rightarrow 7\tan^2 x+8\tan x+1=0$ | M1 | 3.1a |
| | $\tan x=-1,\ -\frac{1}{7}\Rightarrow x=\ldots$ | M1 | 1.1b |
| | $x=-\frac{\pi}{4},\ -0.142$ | A1<br>A1 | 1.1b<br>1.1b |
| | | **(4)** | |
| | | | **(7 marks)** |

**Notes**

(a)

M1: Applies $\cos 2\theta = 1 - 2\sin^2\theta$ in the numerator or $\sin 2\theta = 2\sin\theta\cos\theta$ in the denominator

M1: Applies $\cos 2\theta = 1 - 2\sin^2\theta$ in the numerator and $\sin 2\theta = 2\sin\theta\cos\theta$ in the denominator and simplifies to obtain $k\sec^2\theta$

A1: Correct expression

(b)

M1: Makes the connection with part (a), squares the RHS, applies $\sec^2 x = 1+\tan^2 x$ and collects terms to obtain a 3TQ in tan $x$

M1: Solves a 3TQ in tan $x$ and obtains at least 1 value for $x$

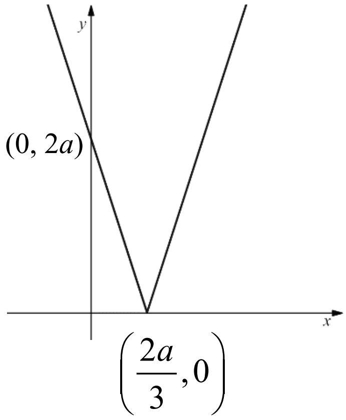
A1: One correct value (allow – 0.785 for $-\frac{\pi}{4}$)

A1: Both correct and no other values in range (allow – 0.785 for $-\frac{\pi}{4}$)

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| 9(a) | $t=0,\ v=56 \Rightarrow 56 = A+B$ **or** $t=5,\ v=10 \Rightarrow 10 = A+Be^{-2.5}$ | M1 | 3.1b |
| | $t=0,\ v=56 \Rightarrow 56 = A+B$ **and** $t=5,\ v=10 \Rightarrow 10 = A+Be^{-2.5}$ **and** $\Rightarrow A = ..., B = ...$ | M1 | 3.1a |
| | At least one of awrt: $A = 5.89$, $B = 50.1$ | A1 | 1.1b |
| | $v = 5.89 + 50.1e^{-0.5t}$ | A1 | 3.3 |
| | | **(4)** | |
| (b) | Minimum $v$ is "5.89" | B1ft | 3.4 |
| | This suggests that the model is appropriate because $5.89 \approx 6$ | B1 | 3.5a |
| | | **(2)** | |
| | | | **(6 marks)** |

**Notes**

(a)
M1: Uses either of the given conditions with the model to obtain at least one equation in $A$ and $B$
M1: Uses both of the given conditions with the model to obtain 2 equations in $A$ and $B$ and solves to obtain values
A1: For $A$ = awrt 5.89 or $B$ = awrt 50.1
A1: Correct equation
(b)
B1ft: Interprets the value of $A$ as the final speed of the skydiver
B1: The model is appropriate as it suggests that the final speed of the skydiver is approximately 6 $\text{ms}^{-1}$

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **10(a)** |  | B1<br><br>B1 | 1.1b<br><br>1.1b |
| | | **(2)** | |
| **(b)** | $3x-2a=x+a \Rightarrow x=...$ or $3x-2a=-x-a \Rightarrow x=...$ | M1 | 1.1b |
| | $3x-2a=x+a \Rightarrow x=...$ and $3x-2a=-x-a \Rightarrow x=...$ | M1 | 2.1 |
| | $x=\frac{a}{4}, \frac{3a}{2}$ | A1 | 1.1b |
| | $\frac{a}{4} \leqslant x \leqslant \frac{3a}{2}$ | A1 | 2.5 |
| | | **(4)** | |
| **(c)** | Maximum value is $5a$ | B1 | 2.2a |
| | $x=\frac{3a}{2} \Rightarrow y=5a-\left\|\frac{1}{2}a-\frac{3a}{2}\right\|=...$ | M1 | 3.1a |
| | $4a \leqslant \mathrm{g}(x) \leqslant 5a$ | A1 | 1.1b |
| | | **(3)** | |
| | | | **(9 marks)** |

**Notes**

(a)
B1: V shape
B1: Fully correct sketch including intercepts
(b)
M1: Attempts to solve one of the equations shown
M1: Considers both possible cases and attempts to solve both of the equations shown
A1: Correct values
A1: Correct range using the correct notation
(c)
B1: Deduces the maximum value
M1: Uses the upper limit from part (b) to find the minimum value
A1: Correct range

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **11(a)** | $339 = ab^{20},\ 414 = ab^{60} \Rightarrow \dfrac{414}{339} = b^{40} \Rightarrow b = \sqrt[40]{\dfrac{414}{339}}$ | M1 | 3.1a |
| | $b = 1.005$ | A1 | 1.1b |
| | $339 = ab^{20} \Rightarrow a = \dfrac{339}{b^{20}}$ or $414 = ab^{60} \Rightarrow a = \dfrac{414}{b^{60}}$ | M1 | 1.1b |
| | $a = 307$ | A1 | 1.1b |
| | | **(4)** | |
| **(b)(i)** | *a* is the concentration in 1960 | B1 | 3.4 |
| **(b)(ii)** | *b* is the factor by which the concentration increases each year | B1 | 3.4 |
| | | **(2)** | |
| **(c)** | $450 = 307 \times 1.005^t \Rightarrow 1.005^t = \dfrac{450}{307}$ | M1 | 3.4 |
| | $1.005^t = \dfrac{450}{307} \Rightarrow t = \log_{1.005}\dfrac{450}{307}$ or $\dfrac{\ln\dfrac{450}{307}}{\ln 1.005}$ | M1 | 1.1b |
| | $t = 76.67...$ | A1 | 1.1b |
| | 2036 or 2037 | A1 | 3.2a |
| | | **(4)** | |
| | | **(10 marks)** | |

**Notes**

(a)
M1: Forms 2 equations in *a* and *b* and solves to obtain a value for *b*.
A1: $b = 1.005$
M1: Uses either equation and their value for *b* to find a value for *a*
A1: $a = 307$
(b)(i)
B1: Correct interpretation for the constant *a*
(b)(ii)
B1: Correct interpretation for the constant *b*
(c)
M1: Uses their values of *a* and *b* and the 450 in the equation for the model and reaches an equation of the form "$1.005$"$^t = k$
M1: For using correct log work to obtain a value for *t*
A1: For awrt 76.7
A1: Interprets the value of *t* correctly and states the year 2036 or 2037, following correct work

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **12(a)** | $\int x\sin kx\,\mathrm{d}x = -\frac{1}{k}x\cos kx + \frac{1}{k}\int \cos kx\,\mathrm{d}x$ | M1<br>A1 | 2.1<br>1.1b |
| | $\frac{1}{k^2}\sin kx - \frac{1}{k}x\cos kx + c$ | A1* | 1.1b |
| | | **(3)** | |
| **(b)** | $\frac{\mathrm{d}H}{\mathrm{d}t} = \frac{t\sin\left(\frac{\pi t}{5}\right)}{10H} \Rightarrow \int 10H\,\mathrm{d}H = \int t\sin\left(\frac{\pi t}{5}\right)\mathrm{d}t$ | M1 | 3.1a |
| | $5H^2 = \frac{25}{\pi^2}\sin\left(\frac{\pi t}{5}\right) - \frac{5}{\pi}t\cos\left(\frac{\pi t}{5}\right) + c$ | M1<br>A1 | 1.1b<br>1.1b |
| | $t = 0, H = 5 \Rightarrow c = 125$ | M1 | 3.4 |
| | $H = \sqrt{\frac{5}{\pi^2}\sin\left(\frac{52\pi}{5}\right) - \frac{1}{\pi}\times 52\cos\left(\frac{52\pi}{5}\right) + 25}$ | M1 | 1.1b |
| | $H = 4.51\,\text{m}$ | A1 | 3.2a |
| | | **(6)** | |
| | | **(9 marks)** | |

**Notes**

(a)

M1: Attempts integration by parts and obtains $\pm Ax\cos kx \pm B\int \cos kx\,\mathrm{d}x$

A1: For $\int x\sin kx\,\mathrm{d}x = -\frac{1}{k}x\cos kx + \frac{1}{k}\int \cos kx\,\mathrm{d}x$

A1*: Correct proof

(b)

M1: Attempts to separate the variables to obtain $\int 10H\,\mathrm{d}H = \int t\sin\left(\frac{\pi t}{5}\right)\mathrm{d}t$ or equivalent e.g.

$\int H\,\mathrm{d}H = \int \frac{1}{10}t\sin\left(\frac{\pi t}{5}\right)\mathrm{d}t$

M1: For using part (a) (or starting again) and integrating both sides to obtain

$AH^2 = B\sin\left(\frac{\pi t}{5}\right) - Ct\cos\left(\frac{\pi t}{5}\right)(+c)$ with or without the " + $c$"

A1: $5H^2 = \frac{25}{\pi^2}\sin\left(\frac{\pi t}{5}\right) - \frac{5}{\pi}t\cos\left(\frac{\pi t}{5}\right)(+c)$ or equivalent with or without the " + $c$"

M1: Substitutes $t = 0$ and $H = 5$ in order to find the constant of integration.

M1: Uses $t = 52$ to find $H$

A1: Correct height of awrt 4.51 m including units.

| Question | Scheme | Marks | AOs |
| --- | --- | --- | --- |
| 13 | $y^2-x^2=8\Rightarrow 2y\frac{\mathrm{d}y}{\mathrm{d}x}-2x=0$ | M1 | 2.1 |
| | $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{x}{y}\Rightarrow\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{y-x\frac{\mathrm{d}y}{\mathrm{d}x}}{y^2}$ | M1<br>A1 | 3.1a<br>1.1b |
| | $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{y-x\frac{\mathrm{d}y}{\mathrm{d}x}}{y^2}\Rightarrow y^3\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=y^2-xy\frac{\mathrm{d}y}{\mathrm{d}x}=y^2-x^2$ | M1 | 3.1a |
| | $\Rightarrow y^3\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=y^2-x^2=8\Rightarrow\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{8}{y^3}$ * | A1* | 2.1 |
| | | **(5)** | |
| | **Alternative 1:** | | |
| | $y^2-x^2=8\Rightarrow 2y\frac{\mathrm{d}y}{\mathrm{d}x}-2x=0$ | M1 | 2.1 |
| | $2y\frac{\mathrm{d}y}{\mathrm{d}x}-2x=0\Rightarrow\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2+y\frac{\mathrm{d}^2y}{\mathrm{d}x^2}-1=0$ | M1<br>A1 | 3.1a<br>1.1b |
| | $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2+y\frac{\mathrm{d}^2y}{\mathrm{d}x^2}-1=0\Rightarrow y^3\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=y^2-y^2\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2=y^2-x^2$ | M1 | 3.1a |
| | $\Rightarrow y^3\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=y^2-x^2=8\Rightarrow\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{8}{y^3}$ * | A1* | 2.1 |
| | | **(5)** | |
| | **Alternative 2:** | | |
| | $y^2-x^2=8\Rightarrow 2y\frac{\mathrm{d}y}{\mathrm{d}x}-2x=0$<br>or<br>$y=\sqrt{x^2+8}\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=x\left(x^2+8\right)^{-\frac{1}{2}}$ | M1 | 2.1 |
| | $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{x}{y}=\frac{x}{\sqrt{x^2+8}}\Rightarrow\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{\sqrt{x^2+8}-x^2\left(x^2+8\right)^{-\frac{1}{2}}}{x^2+8}$ | M1<br>A1 | 3.1a<br>1.1b |
| | $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{x^2+8-x^2}{\left(x^2+8\right)^{\frac{3}{2}}}$ | M1 | 3.1a |
| | $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{8}{y^3}$ * | A1* | 2.1 |
| | | **(5)** | |
| | | | **(5 marks)** |

| Notes |
| --- |
| M1: Adopts a correct strategy of implicit differentiation to obtain $\alpha y\dfrac{dy}{dx}-\beta x=0$<br><br>M1: Rearranges and then applies the quotient rule to obtain $\dfrac{d^2y}{dx^2}=\dfrac{\alpha y-\beta x\dfrac{dy}{dx}}{y^2}$<br>A1: Fully correct differentiation involving the second derivative<br>M1: A complete strategy using the given equation and the first derivative to express the second derivative as an expression not involving the first derivative<br>A1*: Correct proof with no errors<br>**Alternative 1:**<br>M1: Adopts a correct strategy of implicit differentiation to obtain $\alpha y\dfrac{dy}{dx}-\beta x=0$<br>M1: Differentiates implicitly again using the product rule to obtain $\alpha\left(\dfrac{dy}{dx}\right)^2+\beta y\dfrac{d^2y}{dx^2}+k=0$<br>A1: Fully correct differentiation involving the second derivative<br>M1: A complete strategy using the given equation and the first derivative to express the second derivative as an expression not involving the first derivative<br>A1*: Correct proof with no errors<br>**Alternative 2:**<br>M1: Adopts a correct strategy of implicit differentiation to obtain $\alpha y\dfrac{dy}{dx}-\beta x=0$ or expresses $y$ explicitly in terms of $x$ and applies the chain rule<br>M1: Differentiates again using the quotient rule<br>A1: Fully correct differentiation<br>M1: Multiplies numerator and denominator by $\left(x^2+8\right)^{\frac{1}{2}}$<br>A1*: Correct proof with no errors |

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **14(i)** | Let the consecutive odd integers be $2n-1$ and $2n+1$ <br> $(2n-1)^2+(2n+1)^2=4n^2-4n+1+4n^2+4n+1=...$ | M1 | 2.1 |
| | $=8n^2+2$ | A1 | 1.1b |
| | So $(2n-1)^2+(2n+1)^2$ is always 2 more than a multiple of 8 | A1 | 2.4 |
| | | **(3)** | |
| **(ii)** | Assume that $\log_2 5$ is rational so that $\log_2 5=\frac{a}{b}$ <br> where $a$ and $b$ are integers | M1 | 2.4 |
| | $\log_2 5=\frac{a}{b}\Rightarrow 5=2^{\frac{a}{b}}$ | M1 | 1.1b |
| | $5=2^{\frac{a}{b}}\Rightarrow 5^b=2^a$ | A1 | 2.2a |
| | This is a contradiction as a power of 2 cannot equal a power of 5 so $\log_2 5$ must be irrational | A1 | 2.4 |
| | | **(4)** | |
| | | | **(7 marks)** |

**Notes**

(i)
M1: Starts the proof by stating 2 consecutive odd numbers, squares and adds and collects terms
A1: Correct expression
A1: Completes the proof with no errors and an appropriate conclusion
(ii)
M1: Begins the proof by negating the statement e.g. $\log_2 5$ is rational
M1: Applies the definition of logs to eliminate the log
A1: Deduces that $5^b=2^a$
A1: A full and complete argument that completes the contradiction proof