Pearson Edexcel

Mark Scheme

Mock Set 3

Pearson Edexcel GCE Mathematics

Pure 2 Paper 9MA0/02

Edexcel and BTEC Qualifications

Edexcel and BTEC qualifications are awarded by Pearson, the UK's largest awarding body. We provide a wide range of qualifications including academic, vocational, occupational and specific programmes for employers. For further information visit our qualifications websites at www.edexcel.com or www.btec.co.uk. Alternatively, you can get in touch with us using the details on our contact us page at www.edexcel.com/contactus.

Pearson: helping people progress, everywhere

Pearson aspires to be the world's leading learning company. Our aim is to help everyone progress in their lives through education. We believe in every kind of learning, for all kinds of people, wherever they are in the world. We've been involved in education for over 150 years, and by working across 70 countries, in 100 languages, we have built an international reputation for our commitment to high standards and raising achievement through innovation in education. Find out more about how we can help you and your students at: www.pearson.com/uk

Autumn 2021
Publications Code 9MA0_02_MS3_MS

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS
General Instructions for Marking

1. The total number of marks for the paper is 100.

2. The Edexcel Mathematics mark schemes use the following types of marks:
   - **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
   - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - **B** marks are unconditional accuracy marks (independent of M marks)
   - Marks should not be subdivided.

3. Abbreviations
   These are some of the traditional marking abbreviations that will appear in the mark schemes.

   - bod – benefit of doubt
   - ft – follow through
   - the symbol $\sqrt{}$ will be used for correct ft
   - cao – correct answer only
   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - oe – or equivalent (and appropriate)
   - dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - ∗ The answer is printed on the paper
   - ⎣ The second mark is dependent on gaining the first mark

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses <u>and indicates which response they wish to submit</u>, examiners should mark this response.
   If there are several attempts at a question <u>which have not been crossed out</u>, examiners should mark the final answer which is the answer that is the <u>most complete</u>.

6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

General Principles for Further Pure Mathematics Marking

(*But note that specific mark schemes may sometimes override these general principles*)

<u>Method mark for solving 3 term quadratic:</u>

1. Factorisation

$(x^2+bx+c)=(x+p)(x+q)$, where $|pq|=|c|$, leading to $x=\ldots$

$(ax^2+bx+c)=(mx+p)(nx+q)$, where $|pq|=|c|$ and $|mn|=|a|$, leading to $x=\ldots$

2. Formula

Attempt to use the correct formula (with values for *a*, *b* and *c*)

3. Completing the square

Solving $x^2+bx+c=0$: $\left(x\pm\frac{b}{2}\right)^2\pm q\pm c=0,\quad q\neq 0$, leading to $x=\ldots$

<u>Method marks for differentiation and integration:</u>

1. Differentiation

Power of at least one term decreased by 1. $(x^n\rightarrow x^{n-1})$

2. Integration

Power of at least one term increased by 1. $(x^n\rightarrow x^{n+1})$

<u>Use of a formula</u>

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

<u>Method mark</u> for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is <u>not</u> quoted, the method mark can be gained by implication from <u>correct</u> working with values but may be lost if there is any mistake in the working.

<u>Exact answers</u>

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Question	Scheme	Marks	AOs
1	$x^n \to x^{n+1}$	M1	1.1b
	$\int \left(x^4 - 6x^2 + 7\right) \mathrm{d}x = \frac{x^5}{5} - \frac{6x^3}{3} + 7x\left(+c\right)$	A1	1.1b
	$\frac{x^5}{5} - 2x^3 + 7x + c$	A1	1.1b
		(3)	
			(3 marks)

Notes
M1: For raising any power by 1 A1: For 2 correct terms (simplified or unsimplified) A1: All correct and simplified and on one line including + *c*

Question	Scheme	Marks	AOs
2(a)(i)	$u_2 = 3k - 2$	M1	1.1b
(ii)	$u_3 = 3(3k-2) - 2 = 9k - 6 - 2 = 9k - 8$	A1	1.1b
		(2)	
(b)	$u_4 = 3(9k-8) - 2$	M1	1.1b
	$\sum_{r=1}^{4} u_r = 44 \Rightarrow k + 3k - 2 + 9k - 8 + 27k - 26 = 44 \Rightarrow k = ...$	M1	3.1a
	$k = 2$	A1	1.1b
		(3)	
			(5 marks)

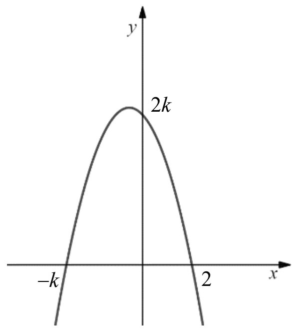
Notes

(a)(i)(ii)
M1: Evidence of use of the given formula to find either u_2 or u_3
A1: Both correct simplified expressions
(b)
M1: Attempts to find the 4th term
M1: A complete method to find *k*: Attempts to find the 4th term, adds their first 4 terms, sets equal to 44 and solves a linear equation in *k*.
A1: Correct value for *k*.

Question	Scheme	Marks	AOs
3(a)	$-3x^2$ should be $(+)3x^2$	B1	2.3
		(1)	
(b)	$x^2 - 27x + 3x^2 + 162 - 36x + 2x^2 = 0$ $\Rightarrow 6x^2 - 63x + 162 = 0$	M1	1.1b
	$2x^2 - 21x + 54 = 0 \Rightarrow (2x-9)(x-6) = 0 \Rightarrow x = ...$	M1	1.1b
	$x = \frac{9}{2}, 6$	A1	1.1b
	$y = \frac{9}{2}, 3$	A1	1.1b
		(4)	
			(5 marks)

Notes

(a)
B1: Identifies the error in the solution
(b)
M1: Proceeds to a 3 term quadratic in either variable, either applies the correction or starts again
M1: Solves their 3TQ
A1: Correct x values
A1: Correct y values

Question	Scheme	Marks	AOs
4(a)		B1 B1 B1	1.1b 1.1b 2.2a
		(3)	
(b)	Correct strategy for x leading to $x = \ldots$	M1	3.1a
	$y = \left(1 - \frac{k}{2} + k\right)\left(2 - 1 + \frac{k}{2}\right)$	M1	1.1b
	$x = \frac{2-k}{2} \quad y = \left(1 + \frac{k}{2}\right)^2$ o.e.	A1	1.1b
		(3)	
			(6 marks)

Notes

(a)
B1: Correct shape 
B1: Correct x intercepts or correct y intercepts
B1: Fully correct diagram with correct intercepts and with the maximum in quadrant 2
(b)
M1: Correct strategy for x coordinate of the stationary point. May be found by calculus e.g. $\frac{dy}{dx} = 2 - 2x - k = 0 \Rightarrow x = \frac{2-k}{2}$ **or** by completing the square **or** by symmetry
M1: Correct attempt to find the y coordinate
A1: Correct coordinates

Question	Scheme	Marks	AOs
5(a)	$\pm\overrightarrow{AB} = \pm\left(7\mathbf{i}+\mathbf{j}+2\mathbf{k}-\left(5\mathbf{i}+3\mathbf{j}-2\mathbf{k}\right)\right)$ $\Rightarrow \left\|\overrightarrow{AB}\right\| = \sqrt{2^2+(-2)^2+4^2}$ or $\Rightarrow \left\|\overrightarrow{AB}\right\|^2 = 2^2+(-2)^2+4^2$	M1	1.1b
	$\left\|\overrightarrow{AB}\right\| = 2\sqrt{6}$	A1	1.1b
		(2)	
(b)	$\overrightarrow{OD} = \overrightarrow{OC}+\overrightarrow{BA} = 4\mathbf{i}+8\mathbf{j}-3\mathbf{k}-2\mathbf{i}+2\mathbf{j}-4\mathbf{k}$	M1	1.1b
	$\overrightarrow{OD} = 2\mathbf{i}+10\mathbf{j}-7\mathbf{k}$	A1	1.1b
		(2)	
(c)	$\overrightarrow{OE} = \overrightarrow{OA}+\frac{3}{2}\overrightarrow{AC}$ or $\overrightarrow{OE} = \overrightarrow{OC}+\frac{1}{2}\overrightarrow{AC}$	M1	3.1a
	E is (3.5, 10.5, −3.5)	A1	1.1b
		(2)	
		(6 marks)	

Notes

(a)
M1: Subtracts either way round and applies Pythagoras to find $\left|\overrightarrow{AB}\right|$ or $\left|\overrightarrow{AB}\right|^2$
A1: For $2\sqrt{6}$
(b)
M1: Correct strategy to find the position vector of *D*
A1: Correct vector
(c)
M1: Interprets the given ratio correctly and then adopts a correct approach to find the coordinates of the point *E*
A1: Correct coordinates and no other coordinates

Question	Scheme	Marks	AOs
6(a)	Area of triangle is $\frac{1}{2}\times 12^2\sin\theta$ Area of segment is $\frac{1}{2}\times 12^2\times\theta-\frac{1}{2}\times 12^2\sin\theta$ $\frac{3}{2}\times 12^2\times\theta-\frac{3}{2}\times 12^2\sin\theta=\frac{1}{2}\times 12^2\sin\theta$	M1	2.1
	$\Rightarrow 3\theta-4\sin\theta=0$ *	A1*	1.1b
		(2)	
(b)	$f'(\theta)=3-4\cos\theta$	B1	1.1b
	$\theta_1=1.2\Rightarrow\theta_2=1.2-\frac{f(1.2)}{f'(1.2)}=1.2-\frac{3.6-4\sin 1.2}{3-4\cos 1.2}=...$	M1	1.1b
	= 1.283	A1	1.1b
		(3)	
			(5 marks)

Notes

(a)
M1: Fully correct strategy using the given information
A1*: Correct result from correct working
(b)
B1: Correct derivative
M1: Applies the Newton-Raphson method correctly
A1: Correct value

Question	Scheme	Marks	AOs
7(a)(i)	$h = 3\,\text{m}\times 0.6^2 = 1.08\,\text{m}$ or e.g. $h = 5\,\text{m}\times 0.6^3 = 1.08\,\text{m}$	B1	2.1
(a)(ii)	$d = 5 + 2(3 + 1.8 + 1.08 + 0.648)$ or $d = 5 + 2\dfrac{3(1-0.6^4)}{1-0.6}$	M1	3.4
	= 18.056 m	A1	1.1b
		(3)	
(b)	$D = 5 + 2\left(\dfrac{3}{1-0.6}\right)$	M1	3.1b
	= 20	A1	1.1b
		(2)	
(c)	e.g. • The model predicts that the ball will continue to bounce indefinitely when in reality it will stop bouncing after a certain number of bounces so the total distance travelled will be less than 20 m. • The diameter of the ball has not been taken into consideration • There could be some horizontal motion • There may be air resistance	B1	3.5b
		(1)	
			(6 marks)

Notes

(a)(i)
B1: Correct explanation
(a)(ii)
M1: Applies a correct strategy for the distance either by adding terms or using the GP sum formula
A1: For awrt 18.1 m
(b)
M1: Recognises the infinite geometric series and applies the sum to infinity formula and adds 5
A1: Correct value
(c)
B1: Makes a suitable comment - see scheme for some possible responses

Question	Scheme	Marks	AOs
8(a)	$R=\sqrt{10}$	B1	1.1b
	$\tan\alpha=\frac{1}{3}\Rightarrow\alpha=...$	M1	1.1b
	$\alpha=0.322$	A1	1.1b
		(3)	
(b)(i)	$6.5-\sqrt{10}$ or awrt 3.34	B1ft	2.2a
(ii)	$\frac{\pi t}{13}-4-0.322=-\pi\Rightarrow t=...$	M1	3.1b
	t = awrt 4.88	A1	1.1b
	4:53 or 4hrs 53 minutes after midnight	A1	3.2a
		(4)	
(c)	$\theta=6.5+\sqrt{10}\cos\left(\frac{\pi t}{13}-4.322\right)$ $\Rightarrow\frac{\mathrm{d}\theta}{\mathrm{d}t}=-\frac{\pi\sqrt{10}}{13}\sin\left(\frac{\pi t}{13}-4.322\right)=-\frac{\pi\sqrt{10}}{13}\sin\left(\frac{\pi(12)}{13}-4.322\right)=...$ **or** $\theta=6.5+3\cos\left(\frac{\pi t}{13}-4\right)+\sin\left(\frac{\pi t}{13}-4\right)$ $\Rightarrow\frac{\mathrm{d}\theta}{\mathrm{d}t}=-\frac{3\pi}{13}\sin\left(\frac{\pi t}{13}-4\right)+\frac{\pi}{13}\cos\left(\frac{\pi t}{13}-4\right)$ $=-\frac{3\pi}{13}\sin\left(\frac{12\pi}{13}-4\right)+\frac{\pi}{13}\cos\left(\frac{12\pi}{13}-4\right)$	M1	3.1b
	= 0.756°C per hour	A1	3.2a
		(2)	
			(9 marks)

Notes

(a)
B1: Correct exact value
M1: Correct strategy to find α
A1: Awrt 0.322
(b)(i)
B1ft: $6.5-\sqrt{10}$ or awrt 3.34 or follow through their R
(b)(ii)
M1: Solves $\frac{\pi t}{13}-4-0.322=\pm\pi$ to reach a value for t
A1: For t = awrt 4.88
A1: Correct time in hours and minutes. Accept either format as shown.
(c)
M1: For the correct strategy to find the rate when t = 12. This requires an attempt to differentiate followed by the substitution of t = 12.
A1: Awrt 0.756°C per hour

Question	Scheme	Marks	AOs
9(a)	$k = -4$ or $x > -4$	B1	2.2a
		(1)	
(b)	$\frac{d}{dx}\left[\ln(x+4)\right] = \frac{1}{(x+4)}$	B1	1.2
	$\frac{d}{dx}\left[\frac{(x+5)(x+1)}{(x+4)}\right] = \frac{(x+4)(2x+6)-(x+5)(x+1)}{(x+4)^2}$	M1 A1	1.1b 1.1b
	$f'(x) = \frac{2x^2+14x+24-x^2-6x-5-x-4}{(x+4)^2}$ $= \frac{x^2+7x+15}{(x+4)^2}$	A1	2.1
		(4)	
(c)	$b^2-4ac = 49-4\times15 = -11 < 0 \Rightarrow x^2+7x+15 > 0$ or $x^2+7x+15 = (x+3.5)^2 - 3.5^2 + 15 = (x+3.5)^2 + 2.75 \Rightarrow x^2+7x+15 > 0$	M1	2.1
	The numerator and denominator are both > 0 Therefore $f'(x) > 0 \Rightarrow$ f is increasing	A1	2.4
		(2)	
			(7 marks)

Notes

(a)
B1: Deduces the correct value for k
(b)
B1: Recalls the correct derivative of $\ln(x + 4)$
M1: For the correct application of the quotient rule
A1: Correct differentiation for the fraction
A1: Fully correct expression
(c)
M1: Considers the discriminant of the numerator or e.g. completes the square in order to show numerator is positive
A1: Suitable conclusion following fully correct work and refers to numerator and denominator being positive

Question	Scheme	Marks	AOs
10(a)	$(36, 27)$	B1	1.1b
		(1)	
(b)	$\frac{dy}{dx} = \frac{6t}{3t^2+3} = \frac{18}{30}$	M1	1.1b
	$y - 27 = \frac{3}{5}(x - 36)$	M1	2.1
	$3x - 5y + 27 = 0$ *	A1*	1.1b
		(3)	
(c)	$3x - 5y + 27 = 0 \Rightarrow 3(t^3 + 3t) - 5(3t^2) + 27 = 0$	M1	3.1a
	$3t^3 - 15t^2 + 9t + 27 = 0 \Rightarrow (t-3)^2(3t+3) = 0$	M1	1.1b
	$t = -1 \Rightarrow Q$ is $(-4, 3)$	A1	2.2a
		(3)	
(d)	$(\text{Area} =) \int y\,dx = \int 3t^2(3t^2+3)\,dt$	M1	2.1
	$= 9\left[\frac{t^5}{5} + \frac{t^3}{3}\right]_{-1}^{3} = 9\left(\frac{3^5}{5} + \frac{3^5}{5} - \left(-\frac{1}{5} - \frac{1}{3}\right)\right)\left(= \frac{2616}{5}\right)$	M1	1.1b
	Area of trapezium = $\frac{1}{2}(36+4)(27+3)(=600)$	M1	2.1
	Area of R is $600 - \frac{2616}{5}$	M1	3.1a
	$= \frac{384}{5}$	A1	1.1b
		(5)	
			(12 marks)

Notes

(a)

B1: Correct coordinates

(b)

M1: Correct strategy for the gradient at *P*

M1: For using their gradient at *P* and their point *P* with a correct straight line method

A1*: Correct equation following correct working

(c)

M1: Awarded for starting the process to find the value of t at Q. E.g. substitutes the parametric form for C into their l

M1: Deduces that $(x - 3)^2$ (or $(x - 3)$) is a factor and uses this to make progress in finding the required linear factor of the cubic. Alternatively solves cubic using calculator.

A1: Deduces the correct coordinates of Q

(d)

M1: For attempting $\int y \times \frac{dx}{dt}\,dt$

M1: Correct use of limits

M1: For the correct trapezium area approach using their values

M1: For a complete strategy for finding the area of R. There must have been an attempt at the area under the curve and an attempt and the trapezium and an attempt to subtract.

A1: Correct area oe e.g. 76.8

Question	Scheme	Marks	AOs
11(a)	$\frac{dx}{dt} = -kx^2$	M1	3.3
	$\frac{dx}{dt} = -kx^2 \Rightarrow \int \frac{dx}{x^2} = \int -k\,dt \Rightarrow \ldots$	M1	2.1
	$\frac{1}{x} = kt + c$	A1	1.1b
	$x = 3.5,\ t = 0 \quad x = 2,\ t = 1 \Rightarrow c = \ldots,\ k = \ldots$	M1	3.1a
	$\frac{1}{x} = \frac{3}{14}t + \frac{2}{7}$ or $t = \frac{1}{kx} + c$	A1	1.1b
	$x = \frac{14}{3t+4}$ *	A1*	2.1
		(6)	
(b)	$0.5 = \frac{14}{3T+4} \Rightarrow 1.5T + 2 = 14 \Rightarrow T = \ldots$	M1	3.4
	$T = 8$	A1	1.1b
		(2)	
			(8 marks)

Notes

(a)

M1: Translates the description of the model into mathematics. Allow $\frac{dx}{dt} = kx^2$

M1: Separates the variables and attempts to integrate.
A1: Correct equation with or without the "+ c"
M1: Uses both conditions in order to find both constants.
A1: Correct equation in any form.
A1*: Fully correct proof.

(b)

M1: Uses $x = 0.5$ in the model and rearranges to find T
A1: Obtains the correct value for T (or states 8 weeks)

Question	Scheme	Marks	AOs
12(a)	$y = a^x \Rightarrow \ln y = \ln a^x = x\ln a$ or $y = a^x \Rightarrow y = \left(e^{\ln a}\right)^x = e^{x\ln a}$	M1	3.1a
	$\ln y = x\ln a \Rightarrow \frac{1}{y}\frac{dy}{dx} = \ln a$ or $y = e^{x\ln a} \Rightarrow \frac{dy}{dx} = e^{x\ln a}\ln a$	M1	1.1b
	$\frac{1}{y}\frac{dy}{dx} = \ln a \Rightarrow \frac{dy}{dx} = y\ln a = a^x \ln a$* or $\frac{dy}{dx} = e^{x\ln a}\ln a = \left(e^{\ln a}\right)^x \ln a = a^x \ln a$* or $\frac{dy}{dx} = e^{x\ln a}\ln a = y\ln a = a^x \ln a$*	A1*	2.1
		(3)	
(b)	$\int 4^x \, dx = \frac{4^x}{\ln 4}(+c)$	B1	2.2a
	$\int_1^2 4^x \, dx = \left[\frac{4^x}{\ln 4}\right]_1^2 = \frac{16}{\ln 4} - \frac{4}{\ln 4} = \frac{12}{\ln 4}$	M1	1.1b
	$\frac{12}{\ln 4} = \frac{12}{2\ln 2} = \frac{6}{\ln 2} = 6(\ln 2)^{-1}$	A1	2.1
		(3)	
			(6 marks)

Notes

(a)

M1: For making the key step of taking ln's and applying the power law of logs to expressing $\ln y$ in terms of x or expresses a as $e^{\ln a}$ and applies the power law of indices

M1: Differentiates implicitly or explicitly for their chosen method

A1*: Fully correct proof

(b)

B1: Deduces the correct integration

M1: Applies the given limits correctly and attempts to combine terms

A1: Correct answer using correct log work

Question	Scheme	Marks	AOs
13(a)	$R = aM^b \Rightarrow \log_{10} R = \log_{10} a + \log_{10} M^b$	M1	2.1
	$\Rightarrow \log_{10} R = \log_{10} a + b\log_{10} M$ *	A1*	1.1b
		(2)	
(b)	$b = 0.636$	B1	2.2a
	$1.2 = "0.636" \times 0.7 + \log_{10} a \Rightarrow a = 10^{0.754...}$ or $1.9 = "0.636" \times 1.8 + \log_{10} a \Rightarrow a = 10^{0.755...}$	M1	3.1a
	$R = 5.68M^{0.636}$	A1	3.3
		(3)	
(c)	The resting metabolic rate for a mammal of mass 1 g	B1	3.2a
		(1)	
			(6 marks)

Notes

(a)
M1: Takes logs of both sides and shows the addition law
A1*: Uses the power law to obtain the given equation
(b)
B1: Deduces the correct value for *b* (Allow awrt 0.636 or exact $\frac{7}{11}$)
M1: Correct strategy to find the value of *a*
A1: Correct equation. Allow 5.68 or 5.69 for *a*.
(c)
B1: Correct interpretation

Question	Scheme	Marks	AOs
14(a)	$u = 1 + \sin^2 x \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = 2\sin x \cos x$	B1	1.1b
	$\int \frac{8\tan x}{1+\sin^2 x}\mathrm{d}x = \int \frac{8\tan x}{u}\frac{\mathrm{d}u}{2\sin x\cos x}$	M1	2.1
	$= \int \frac{4}{u\cos^2 x}\mathrm{d}u = \int \frac{4}{u\left(1-\sin^2 x\right)}\mathrm{d}u = \int \frac{4}{u\left(1-\left(u-1\right)\right)}\mathrm{d}u$	M1	3.1a
	$= \int \frac{4}{u\left(2-u\right)}\mathrm{d}u$ *	A1*	2.1
	$p = 1$ and $q = \frac{5}{4}$	B1	2.2a
		(5)	
(b)	$\frac{4}{u\left(2-u\right)} \equiv \frac{a}{u} + \frac{b}{2-u} \Rightarrow a = ..,b = ...$	M1	2.1
	$\frac{4}{u\left(2-u\right)} \equiv \frac{2}{u} + \frac{2}{2-u}$	A1	1.1b
	$= \int \frac{4}{u\left(2-u\right)}\mathrm{d}u = 2\ln u - 2\ln\left(2-u\right)\left(+c\right)$	dM1 A1ft	3.1a 1.1b
	$= \int_1^{\frac{5}{4}} \frac{4}{u\left(2-u\right)}\mathrm{d}u = \left[2\ln u - 2\ln\left(2-u\right)\right]_1^{\frac{5}{4}} = 2\ln\frac{5}{4} - 2\ln\left(2-\frac{5}{4}\right) - \left(0\right)$	M1	1.1b
	$= \ln\frac{25}{9}$	A1	2.1
		(6)	
(11 marks)			
Notes			

Question	Scheme	Marks	AOs
15	$\ln\left(3^x-1\right)-\ln 3=\ln\left(3^x+5\right)-\ln\left(3^x-1\right)$ $\dfrac{\left(3^x-1\right)}{3}=\dfrac{\left(3^x+5\right)}{\left(3^x-1\right)}$	M1	3.1a
	$\left(3^x-1\right)^2=3\left(3^x+5\right)\Rightarrow\left(3^x\right)^2-5\left(3^x\right)-14=0$	M1 A1	2.1 1.1b
	$\left(3^x\right)^2-5\left(3^x\right)-14=0\Rightarrow 3^x=7\Rightarrow x=...$	M1	1.1b
	$3^x=7\Rightarrow x=\log_3 7$ o.e. only	A1	2.3
		(5)	
		(5 marks)	

Notes

M1: Recognises the arithmetic sequence property to form an equation connecting the terms and then applies the subtraction rule of logarithms to eliminate the ln's

M1: Forms a 3TQ equation in 3^x

A1: Correct 3TQ

M1: Solves their 3TQ in 3^x and solves for x using logs appropriately, ignore any reference to -2

A1: Cao. Allow equivalent exact answers e.g. $\dfrac{\ln 7}{\ln 3}, \dfrac{\log 7}{\log 3}$ and no other solutions