

Paper Reference(s)

**6665/01**

# Edexcel GCE

## Core Mathematics C3

### Silver Level S3

**Time: 1 hour 30 minutes****Materials required for examination papers**

Mathematical Formulae (Green)

**Items included with question**

Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

**Instructions to Candidates**

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Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C3), the paper reference (6665), your surname, initials and signature.

**Information for Candidates**

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A booklet 'Mathematical Formulae and Statistical Tables' is provided.

Full marks may be obtained for answers to ALL questions.

There are 8 questions in this question paper. The total mark for this paper is 75.

**Advice to Candidates**

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You must ensure that your answers to parts of questions are clearly labelled.

You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

**Suggested grade boundaries for this paper:**

<b>A*</b>	<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>	<b>E</b>
<b>70</b>	<b>63</b>	<b>56</b>	<b>48</b>	<b>42</b>	<b>38</b>

1. (a) Find the value of  $\frac{dy}{dx}$  at the point where  $x = 2$  on the curve with equation

$$y = x^2 \sqrt{(5x - 1)}. \quad (6)$$

- (b) Differentiate  $\frac{\sin 2x}{x^2}$  with respect to  $x$ .

(4)

January 2009

2.  $f(x) = x^3 + 2x^2 - 3x - 11$

- (a) Show that  $f(x) = 0$  can be rearranged as

(2)

$$x = \sqrt{\left(\frac{3x + 11}{x + 2}\right)}, \quad x \neq -2.$$

The equation  $f(x) = 0$  has one positive root  $\alpha$ .

The iterative formula  $x_{n+1} = \sqrt{\left(\frac{3x_n + 11}{x_n + 2}\right)}$  is used to find an approximation to  $\alpha$ .

- (b) Taking  $x_1 = 0$ , find, to 3 decimal places, the values of  $x_2$ ,  $x_3$  and  $x_4$ .

(3)

- (c) Show that  $\alpha = 2.057$  correct to 3 decimal places.

(3)

January 2010

3. (a) Express

$$\frac{4x-1}{2(x-1)} - \frac{3}{2(x-1)(2x-1)}$$

as a single fraction in its simplest form.

(4)

Given that

$$f(x) = \frac{4x-1}{2(x-1)} - \frac{3}{2(x-1)(2x-1)} - 2, \quad x > 1,$$

(b) show that

$$f(x) = \frac{3}{2x-1}.$$

(2)

(c) Hence differentiate  $f(x)$  and find  $f'(2)$ .

(3)

**January 2011**

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4.

$$f(x) = 7\cos x + \sin x$$

Given that  $f(x) = R\cos(x - a)$ , where  $R > 0$  and  $0 < a < 90^\circ$ ,

(a) find the exact value of  $R$  and the value of  $a$  to one decimal place.

(3)

(b) Hence solve the equation

$$7\cos x + \sin x = 5$$

for  $0 \leq x < 360^\circ$ , giving your answers to one decimal place.

(5)

(c) State the values of  $k$  for which the equation

$$7\cos x + \sin x = k$$

has only one solution in the interval  $0 \leq x < 360^\circ$ .

(2)

**June 2013 (R)**

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5. The function  $f$  is defined by

$$f: x \mapsto 4 - \ln(x + 2), \quad x \in \mathbb{R}, \quad x \geq -1.$$

(a) Find  $f^{-1}(x)$ . **(3)**

(b) Find the domain of  $f^{-1}$ . **(1)**

The function  $g$  is defined by

$$g: x \mapsto e^{x^2} - 2, \quad x \in \mathbb{R}.$$

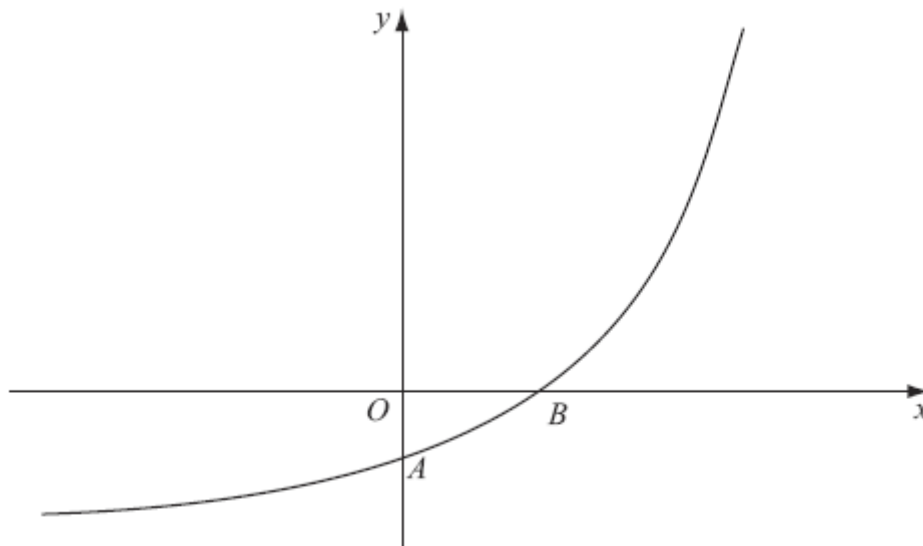
(c) Find  $fg(x)$ , giving your answer in its simplest form. **(3)**

(d) Find the range of  $fg$ . **(1)**

**June 2011**

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6.



**Figure 1**

Figure 1 shows a sketch of part of the curve with equation  $y = f(x)$ ,  $x \in \mathbb{R}$ .

The curve meets the coordinate axes at the points  $A(0, 1 - k)$  and  $B(\frac{1}{2} \ln k, 0)$ , where  $k$  is a constant and  $k > 1$ , as shown in Figure 2.

On separate diagrams, sketch the curve with equation

(a)  $y = |f(x)|$ , (3)

(b)  $y = f^{-1}(x)$ . (2)

Show on each sketch the coordinates, in terms of  $k$ , of each point at which the curve meets or cuts the axes.

Given that  $f(x) = e^{2x} - k$ ,

(c) state the range of  $f$ , (1)

(d) find  $f^{-1}(x)$ , (3)

(e) write down the domain of  $f^{-1}$ . (1)

**June 2009**

7.  $f(x) = x^2 - 3x + 2 \cos\left(\frac{1}{2}x\right), \quad 0 \leq x \leq \pi.$

(a) Show that the equation  $f(x) = 0$  has a solution in the interval  $0.8 < x < 0.9$ . (2)

The curve with equation  $y = f(x)$  has a minimum point  $P$ .

(b) Show that the  $x$ -coordinate of  $P$  is the solution of the equation

$$x = \frac{3 + \sin\left(\frac{1}{2}x\right)}{2}. \quad (4)$$

(c) Using the iteration formula

$$x_{n+1} = \frac{3 + \sin\left(\frac{1}{2}x_n\right)}{2}, \quad x_0 = 2,$$

find the values of  $x_1, x_2$  and  $x_3$ , giving your answers to 3 decimal places. (3)

(d) By choosing a suitable interval, show that the  $x$ -coordinate of  $P$  is 1.9078 correct to 4 decimal places. (3)

**January 2012**

8. The curve  $C$  has equation

$$y = \frac{3 + \sin 2x}{2 + \cos 2x}.$$

(a) Show that

$$\frac{dy}{dx} = \frac{6 \sin 2x + 4 \cos 2x + 2}{(2 + \cos 2x)^2} \quad (4)$$

(b) Find an equation of the tangent to  $C$  at the point on  $C$  where  $x = \frac{\pi}{2}$ .

Write your answer in the form  $y = ax + b$ , where  $a$  and  $b$  are exact constants. (4)

**January 2011**

**TOTAL FOR PAPER: 75 MARKS**

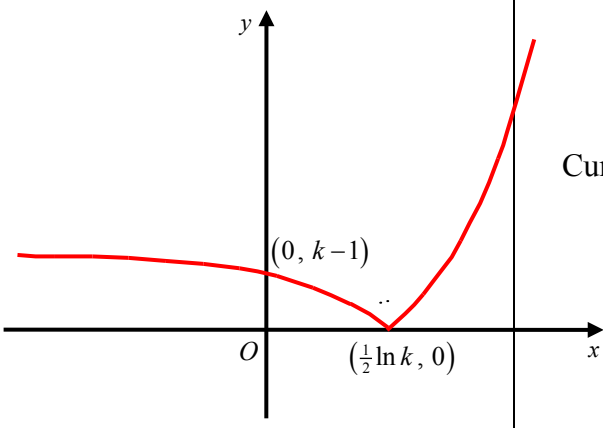
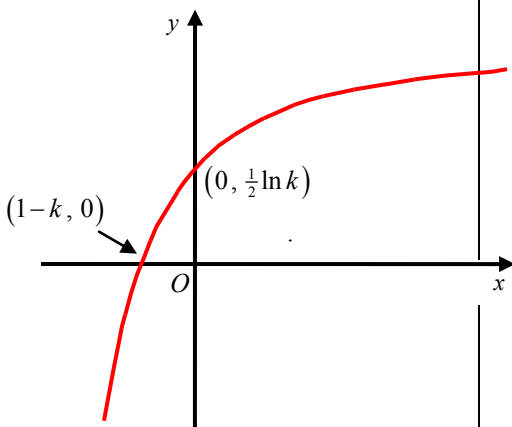
**END**

Question Number	Scheme	Marks
1. (a)	$\frac{d}{dx}(\sqrt{5x-1}) = \frac{d}{dx}((5x-1)^{\frac{1}{2}})$ $= 5 \times \frac{1}{2} (5x-1)^{-\frac{1}{2}}$ $\frac{dy}{dx} = 2x\sqrt{5x-1} + \frac{5}{2}x^2(5x-1)^{-\frac{1}{2}}$ <p>At <math>x = 2</math>, <math>\frac{dy}{dx} = 4\sqrt{9} + \frac{10}{\sqrt{9}} = 12 + \frac{10}{3}</math></p> $= \frac{46}{3}$	<p>M1 A1</p> <p>M1 A1ft</p> <p>M1</p> <p>Accept awrt 15.3 A1 (6)</p>
(b)	$\frac{d}{dx}\left(\frac{\sin 2x}{x^2}\right) = \frac{2x^2 \cos 2x - 2x \sin 2x}{x^4}$	<p>M1, A1+A1 A1 (4) [10]</p>
2.	$f(x) = x^3 + 2x^2 - 3x - 11$	
(a)	$f(x) = 0 \Rightarrow x^3 + 2x^2 - 3x - 11 = 0$ $\Rightarrow x^2(x+2) - 3x - 11 = 0$ $\Rightarrow x^2(x+2) = 3x + 11$ $\Rightarrow x^2 = \frac{3x+11}{x+2}$ $\Rightarrow x = \sqrt{\left(\frac{3x+11}{x+2}\right)}$	<p>M1</p> <p>A1 AG (2)</p>
(b)	<p>Iterative formula: <math>x_{n+1} = \sqrt{\left(\frac{3x_n+11}{x_n+2}\right)}</math>, <math>x_1 = 0</math></p> $x_2 = \sqrt{\left(\frac{3(0)+11}{(0)+2}\right)}$ $x_2 = 2.34520788\dots$ $x_3 = 2.037324945\dots$ $x_4 = 2.058748112\dots$	<p>M1</p> <p>A1</p> <p>A1 (3)</p>
(c)	<p>Let <math>f(x) = x^3 + 2x^2 - 3x - 11 = 0</math></p> $f(2.0565) = -0.013781637\dots$ $f(2.0575) = 0.0041401094\dots$ <p>Sign change (and <math>f(x)</math> is continuous) therefore a root <math>\alpha</math> is such that <math>\alpha \in (2.0565, 2.0575) \Rightarrow \alpha = 2.057</math> (3 dp)</p>	<p>M1 dM1</p> <p>A1 (3) [8]</p>

Question Number	Scheme	Marks
3. (a)	$\frac{4x-1}{2(x-1)} - \frac{3}{2(x-1)(2x-1)}$ $= \frac{(4x-1)(2x-1) - 3}{2(x-1)(2x-1)}$ $= \frac{8x^2 - 6x - 2}{\{2(x-1)(2x-1)\}}$ $= \frac{2(x-1)(4x+1)}{\{2(x-1)(2x-1)\}}$ $= \frac{4x+1}{2x-1}$	<p>An attempt to form a single fraction M1</p> <p>Simplifies to give a correct quadratic numerator over a correct quadratic denominator A1 aef</p> <p>An attempt to factorise a 3 term quadratic numerator M1</p> <p>A1 (4)</p>
(b)	$f(x) = \frac{4x-1}{2(x-1)} - \frac{3}{2(x-1)(2x-1)} - 2, \quad x > 1$ $f(x) = \frac{4x+1}{2x-1} - 2$ $= \frac{4x+1 - 2(2x-1)}{2x-1}$ $= \frac{4x+1 - 4x+2}{2x-1}$ $= \frac{3}{2x-1}$	<p>An attempt to form a single fraction M1</p> <p>Correct result A1 * (2)</p>
(c)	$f(x) = \frac{3}{2x-1} = 3(2x-1)^{-1}$ $f'(x) = 3(-1)(2x-1)^{-2}(2) \quad \pm k(2x-1)^{-2}$ $f'(2) = \frac{-6}{9} = -\frac{2}{3} \quad \text{Either } \frac{-6}{9} \text{ or } -\frac{2}{3}$	<p>M1</p> <p>A1 aef</p> <p>A1 (3)</p> <p><b>[9]</b></p>



Question Number	Scheme	Marks
<p>4. (a)</p> <p>(b)</p> <p>(c)</p>	$7 \cos x + \sin x = R \cos(x - \alpha)$ $R = \sqrt{7^2 + 1^2} = \sqrt{50} = (5\sqrt{2})$ $\alpha = \arctan\left(\frac{1}{7}\right) = 8.13\dots = \text{awrt } 8.1^\circ$ $\sqrt{50} \cos(x - 8.1) = 5 \Rightarrow \cos(x - 8.1) = \frac{5}{\sqrt{50}}$ $x - 8.1 = 45 \Rightarrow x = 53.1^\circ$ <p>AND <math>x - 8.1 = 315 \Rightarrow x = 323.1^\circ</math></p> <p>One solution if <math>\frac{k}{\sqrt{50}} = \pm 1, \Rightarrow k = \pm\sqrt{50}</math> <span style="float: right;">ft on R</span></p>	<p>B1</p> <p>M1A1</p> <p><b>(3)</b></p> <p>M1</p> <p>M1A1</p> <p>M1A1</p> <p><b>(5)</b></p> <p>M1</p> <p>A1ft</p> <p><b>(2)</b></p> <p><b>[10]</b></p>
<p>5. (a)</p> <p>(b)</p> <p>(c)</p> <p>(d)</p>	$y = 4 - \ln(x + 2)$ $\ln(x + 2) = 4 - y$ $x + 2 = e^{4-y}$ $x = e^{4-y} - 2$ $f^{-1}(x) = e^{4-x} - 2$ $x \leq 4$ $fg(x) = 4 - \ln(e^{x^2} - 2 + 2)$ $fg(x) = 4 - x^2$ $fg(x) \leq 4$	<p>M1</p> <p>oe M1A1</p> <p><b>(3)</b></p> <p>B1</p> <p><b>(1)</b></p> <p>M1</p> <p>dM1A1</p> <p><b>(3)</b></p> <p>B1ft</p> <p><b>(1)</b></p> <p><b>[10]</b></p>

Question Number	Scheme	Marks
6. (a)	 <p data-bbox="1018 203 1281 280">Curve retains shape when <math>x &gt; \frac{1}{2} \ln k</math></p> <p data-bbox="906 360 1281 436">Curve reflects through the <math>x</math>-axis when <math>x &lt; \frac{1}{2} \ln k</math></p> <p data-bbox="975 517 1281 638"><math>(0, k-1)</math> and <math>(\frac{1}{2} \ln k, 0)</math> marked in the correct positions.</p>	<p data-bbox="1302 264 1342 297">B1</p> <p data-bbox="1302 421 1342 454">B1</p> <p data-bbox="1302 577 1409 611">B1 (3)</p>
(b)	 <p data-bbox="906 712 1281 862">Correct shape of curve. The curve should be contained in quadrants 1, 2 and 3 (Ignore asymptote)</p> <p data-bbox="975 929 1281 974"><math>(1-k, 0)</math> and <math>(0, \frac{1}{2} \ln k)</math></p>	<p data-bbox="1302 786 1342 819">B1</p> <p data-bbox="1302 920 1409 954">B1 (2)</p>
(c)	<p data-bbox="331 1205 895 1249">Range of <math>f</math>: <math>f(x) &gt; -k</math> or <math>y &gt; -k</math> or <math>(-k, \infty)</math></p>	<p data-bbox="1302 1211 1409 1245">B1 (1)</p>
(d)	<p data-bbox="331 1272 639 1317"><math>y = e^{2x} - k \Rightarrow y + k = e^{2x}</math></p> <p data-bbox="331 1339 544 1384"><math>\Rightarrow \ln(y + k) = 2x</math></p> <p data-bbox="331 1406 544 1451"><math>\Rightarrow \frac{1}{2} \ln(y + k) = x</math></p> <p data-bbox="331 1485 655 1529">Hence <math>f^{-1}(x) = \frac{1}{2} \ln(x + k)</math></p>	<p data-bbox="1302 1279 1342 1312">M1</p> <p data-bbox="1302 1413 1342 1447">M1</p> <p data-bbox="1302 1469 1409 1503">A1 cao</p>
(e)	<p data-bbox="331 1619 799 1664"><math>f^{-1}(x)</math>: Domain: <math>x &gt; -k</math> or <math>(-k, \infty)</math></p>	<p data-bbox="1302 1581 1366 1615">B1ft</p> <p data-bbox="1366 1637 1409 1671">(1)</p> <p data-bbox="1350 1693 1409 1727"><b>[10]</b></p>

Question Number	Scheme	Marks
7. (a)	$f(0.8) = 0.082, f(0.9) = -0.089$ Change of sign $\Rightarrow$ root (0.8,0.9)	M1 A1 (2)
(b)	$f'(x) = 2x - 3 - \sin\left(\frac{1}{2}x\right)$ Sets $f'(x) = 0 \Rightarrow x = \frac{3 + \sin\left(\frac{1}{2}x\right)}{2}$	M1 A1 M1A1* (4)
(c)	Sub $x_0 = 2$ into $x_{n+1} = \frac{3 + \sin\left(\frac{1}{2}x_n\right)}{2}$ $x_1 = \text{awrt } 1.921, x_2 = \text{awrt } 1.91(0) \text{ and } x_3 = \text{awrt } 1.908$	M1 A1, A1 (3)
(d)	[1.90775, 1.90785] $f'(1.90775) = -0.00016... \text{ AND } f'(1.90785) = 0.0000076...$ Change of sign $\Rightarrow x = 1.9078$	M1 M1 A1 (3) [12]

Qu. No.	Scheme	Marks
<p><b>8</b></p> <p>(a) <math>y = \frac{3 + \sin 2x}{2 + \cos 2x}</math></p> <p>Apply quotient rule:</p> $\left\{ \begin{array}{l} u = 3 + \sin 2x \quad v = 2 + \cos 2x \\ \frac{du}{dx} = 2 \cos 2x \quad \frac{dv}{dx} = -2 \sin 2x \end{array} \right\}$ $\frac{dy}{dx} = \frac{2 \cos 2x(2 + \cos 2x) - -2 \sin 2x(3 + \sin 2x)}{(2 + \cos 2x)^2}$ $= \frac{4 \cos 2x + 2 \cos^2 2x + 6 \sin 2x + 2 \sin^2 2x}{(2 + \cos 2x)^2}$ $= \frac{4 \cos 2x + 6 \sin 2x + 2(\cos^2 2x + \sin^2 2x)}{(2 + \cos 2x)^2}$ $= \frac{4 \cos 2x + 6 \sin 2x + 2}{(2 + \cos 2x)^2} \quad (\text{as required})$	<p>Applying <math>\frac{u'v - uv'}{v^2}</math></p> <p>Any one term correct on the numerator</p> <p>Fully correct (unsimplified).</p> <p>For correct proof with an understanding that <math>\cos^2 2x + \sin^2 2x = 1</math>.</p> <p>No errors seen in working.</p>	<p>M1</p> <p>A1</p> <p>A1</p> <p>A1*</p> <p>(4)</p>
<p>(b) When <math>x = \frac{\pi}{2}</math>, <math>y = \frac{3 + \sin \pi}{2 + \cos \pi} = \frac{3}{1} = 3</math></p> <p>At <math>(\frac{\pi}{2}, 3)</math>, <math>m(\mathbf{T}) = \frac{6 \sin \pi + 4 \cos \pi + 2}{(2 + \cos \pi)^2} = \frac{-4 + 2}{1^2} = -2</math></p> <p>Either <math>\mathbf{T}</math>: <math>y - 3 = -2(x - \frac{\pi}{2})</math>  or <math>y = -2x + c</math> and  <math>3 = -2(\frac{\pi}{2}) + c \Rightarrow c = 3 + \pi</math>;</p> <p><math>\mathbf{T}</math>: <math>y = -2x + (\pi + 3)</math></p>	<p><math>y = 3</math></p> <p><math>m(\mathbf{T}) = -2</math></p> <p><math>y - y_1 = m(x - \frac{\pi}{2})</math> with 'their TANGENT gradient' and their <math>y_1</math>;  or uses <math>y = mx + c</math> with 'their TANGENT gradient';</p> <p><math>y = -2x + \pi + 3</math></p>	<p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>(4)</p> <p>[8]</p>

## Examiner reports

### Question 1

This proved a good starting question which tested the basic laws of differentiation; the chain, product and quotient laws. Almost all candidates were able to gain marks on the question. In part (a), most realised that they needed to write  $\sqrt{(5x-1)}$  as  $(5x-1)^{\frac{1}{2}}$  before differentiating.

The commonest error was to give  $\frac{d}{dx}\left((5x-1)^{\frac{1}{2}}\right) = \frac{1}{2}(5x-1)^{-\frac{1}{2}}$ , omitting the factor 5. It was

disappointing to see a number of candidates incorrectly interpreting brackets, writing  $(5x-1)^{\frac{1}{2}} = 5x^{\frac{1}{2}} - 1^{\frac{1}{2}}$ . Not all candidates realised that the product rule was needed and the use of

$\frac{d}{dx}(uv) = \frac{du}{dx} \times \frac{dv}{dx}$  was not uncommon. Part (b) was generally well done but candidates

should be aware of the advantages of starting by quoting a correct quotient rule. The examiner can then award method marks even if the details are incorrect. The commonest error seen was

writing  $\frac{d}{dx}(\sin 2x) = \cos 2x$ . A number of candidates caused themselves unnecessary

difficulties by writing  $\sin 2x = 2 \sin x \cos x$ . Those who used the product rule in part (b) seemed, in general, to be more successful than those who had used this method in other recent examinations.

### Question 2

All three parts of this question were well answered by the overwhelming majority of candidates who demonstrated their confidence with the topic of iteration.

Part (a) was well answered by the majority of candidates although a significant minority of candidates were not rigorous enough in their proof. Some candidates assumed the step of factorising a common factor of  $x^2$  from their first two terms rather than explicitly showing it. A few candidates attempted to reverse the proof and arrived at the correct equation but many of these candidates lost the final accuracy mark by not referring to  $f(x) = 0$ .

Part (b) was almost universally answered correctly, although a few candidates incorrectly gave  $x_4$  as 2.058.

The majority of candidates who attempted part (c) choose an appropriate interval for  $x$  and evaluated  $f(x)$  at both ends of that interval. The majority of these candidates chose the interval  $(2.0565, 2.0575)$  although incorrect intervals, such as  $(2.056, 2.058)$  were seen.

There were a few candidates who chose the interval  $(2.0565, 2.0574)$ . This probably reflects a misunderstanding of the nature of rounding but a change of sign over this interval does establish the correct result and this was accepted for full marks. To gain the final mark, candidates are expected to give a reason that there is a sign change, and give a suitable conclusion such as that the root is 2.057 to 3 decimal places or  $\alpha = 2.057$  or even QED.

A minority of candidates attempted part (c) by using a repeated iteration technique. Almost all of these candidates iterated as far as  $x_6$  (or beyond) but most of these did not write down their answers to at least four decimal places. Of those candidates who did, very few of them managed to give a valid conclusion.

**Question 3**

In part (a) there were many fully correct and well-presented answers. Most candidates were able to combine the two fractions, although some used unnecessarily complex denominators. A number of responses included errors made when cancelling the 2 from the numerator and denominator. A number of responses failed to simplify their answer and lost the final mark. A few candidates, having correctly given the fraction with a common denominator on the first line, cancelled one or more of the brackets leaving both numerator and denominator as linear expressions. A common thread running through the paper for numerous candidates, not just the weaker ones, was the lack of consistent bracketing. Candidates need to be aware that this could lead to the potential loss of many marks.

Part (b), most candidates realised that they could use their answer from part (a) and many were able to successfully demonstrate the proof. A few candidates started from scratch and often went on to gain full marks.

The method used in part (c) was equally split between those deciding to use the chain rule and those using the quotient rule. For the quotient rule, a number differentiated 3 incorrectly, usually as 1. Having found the expression for  $dy/dx$ , a number of candidates multiplied out the denominator in terms of  $x$  first, before substituting in  $x = 2$ . A surprising number reached the correct fractional answer for  $dy/dx$  but, on substituting in  $x = 2$ , gave an answer of  $-6/25$  (stating that  $2 \times 2 - 1 = 5$ ). A small number of candidates decided to differentiate their incorrect answer for (b). Candidates should be advised that this should be avoided at all costs, especially where an answer is given in the paper.

**Question 4**

In part (a), the vast majority produced completely correct solutions. There were very few answers in radians and most realised the requirement to give an exact value for  $R$ . The favoured method for  $\alpha$  was by using  $\tan$  from a full expansion of the addition formula. Errors were made when 7.07 being written without prior evidence of the square root of 50. Occasionally the angle was rounded to 8 degrees, or given as 81.9 degrees (from incorrect  $\tan$  work).

Part (b) was generally answered very well with clear stages of working out shown. Nearly all candidates used (a) and moved to an equation involving  $\cos(x - \alpha)$ . Those candidates who did not use an exact value for  $R$ , rarely worked with a more accurate value than 7.07 (or even 7.1) leading to inaccurate answers. Most could find a second angle (which should have in the fourth quadrant) but a small number looked in error in the third or made no attempt at all. Errors were also seen by subtracting 8.1 instead of adding.

Part (c) was generally poorly answered and it was evident that many candidates did not know how to answer this question. Perhaps the slightly different way of linking the minimum and maximum values with the roots of an equation showed some lack of understanding across the specification.

**Question 5**

A straightforward question testing understanding of log laws and functions. It was rare to get a fully correct answer, however the majority of candidates achieved at least half marks. It is obvious that many students struggle with the concepts of domain and range and how to find them.

In part (a) the vast majority of candidates had sensible ideas about finding the inverse of a function and the first method mark was usually awarded. The responses were approximately

equally split on whether the variables  $x$  and  $y$  were interchanged at the beginning or at the end. They generally set their method out clearly thus gaining the second method mark. Some extremely poor algebra was seen however revealing a lack of understanding of the laws of logarithms.

Examples of this included going from  $(4 - x)$  to  $e^4 - e^x$ . Sign errors were the biggest problem when taking exponentials, particularly for those candidates who tried to deal with  $-\ln(x + 2)$ , leading to  $-(x + 2)$  rather than rearranging the equation to give  $4 - y = \ln(x + 2)$ . A few candidates managed to cope with  $-\ln(x + 2)$  by converting it to  $\ln(x + 2)^{-1}$  and then went on to gain full marks for this part.

Part (b) part caused more problems on the paper than any other part, with only the better candidates achieving the correct domain. There were a variety of numerical values given, but most just stated it as being any real number. When they managed to get 4, some wrote  $y$  or  $f^{-1} \leq 4$  or  $x > 4$ , indicating that the idea of a domain was not fully understood. It was rare for students to spot the link between parts (b) and (d), so that if part (d) was correct, then part (b) was not modified.

In part (c) the composite function was well attempted and simplified. Very few applied the functions the wrong way around. Only a few failed to get the first mark by omitting  $+2$  or  $-2$ . It was fairly common to see candidates getting to  $4 - \ln e^{x^2}$  and stopping, so not gaining any further marks.

Several opted to apply  $\ln$  immediately without cancelling the 2s and got  $+\ln 2 - \ln 2$ . The candidates in part (d) were more successful here than in part (b), but there was little evidence of a realisation that the two answers were linked. A few graphs appeared, but it was surprising the number who correctly answered part (c) but could not establish the range of a negative quadratic. Again, stating the range was any real number or offering incorrect numerical values was common and also sometimes the wrong variable was used, for example  $x$  or  $f(x)$ .

### Question 6

In part (a), almost all candidates realised that the transformed curve was the same as the original curve for  $x > \frac{1}{2}\ln k$ , and only a few failed to reflect the other part of the curve correctly through the  $x$ -axis. A significant number of candidates in this part struggled to write down the correct coordinates for the  $y$ -intercept in terms of  $k$ . The most common incorrect answers were  $(0, k+1)$  and  $(0, 1-k)$ . A few candidates did not state the coordinates of the  $y$ -intercept in the simplest form. An answer of  $(0, |1-k|)$  was accepted on the  $y$ -axis in this part.

In part (b), many candidates realised that they needed to reflect the given curve through the line  $y=x$ . Although the majority of these candidates managed to draw the correct shape of the transformed curve, a few seemed unable to visualise the correct position after reflection and incorrectly positioned their curve going through the fourth quadrant. Those candidates who correctly reflected the original curve through the line  $y=x$  were often unable to see that the effect of reflecting points  $A$  and  $B$  in the line  $y=x$  was a reversal of  $x$ - and  $y$ -coordinates. The most common incorrect coordinates for the  $x$  and  $y$  intercepts in this part were  $(-\frac{1}{2}\ln k, 0)$  and  $(0, -1+k)$  respectively.

In parts (a) and (b), examiners were fairly tolerant with curvature. Those candidates, however, who drew curves going back on themselves to give an upside-down U in the second quadrant in part (a) or a C-shape in the third quadrant in part (b) were not awarded the relevant mark for the shape of their curve.

The majority of candidates struggled with part (c). Some candidates sketched the curve of  $y=e^{2x}$  and proceeded to translate this curve down  $k$  units in the  $y$ -direction and in most cases these candidates were able to write down the correct range. A significant number of candidates wrote their range using  $x$  rather than using either  $y$  or  $f(x)$ . Common incorrect answers for the range were  $y \in \mathbb{R}$ ,  $y > 0$ ,  $y > k$  or  $y \geq k$ .

Part (d) was well answered and a majority of candidates were able to score some marks with a large number scoring full marks. The general procedure of changing the subject and switching  $x$  and  $y$  was well known. There was the occasional difficulty with taking logarithms of both sides. A common error was an incorrect answer of  $\frac{1}{2}\ln(x-k)$ . As  $x$  only appears once in the original function, a few candidates instead chose to use a flowchart method to find the inverse. Nearly all those who did this arrived at the correct answer.

Again, as with part (c), many candidates struggled to give the correct domain for the inverse function in part (e). Those candidates who correctly stated that the domain of the inverse function is the same as the range of the original function failed in many cases to obtain the follow-through mark. This was because they did not change  $y$  or  $f(x)$  in their range inequality to  $x$  in their domain inequality.

### Question 7

This type of question is accessible to most candidates with zero scores being rare.

Part (a): This part was done well on the whole with most candidates picking up both marks. Some candidates gained the first M mark for showing substitution even if both values were incorrect from using their calculators in degree mode. However most candidates correctly calculated  $f(0.8)$  and  $f(0.9)$  and were able to give both a reason and a conclusion to justify their answer. For candidates who used a tighter interval, usually 0.81 and 0.89 most referred back to the original interval in the conclusion which is acceptable.

Part (b) was a subtle variation on the usual theme, with both differentiation and rearrangement required to score full marks. A few unfortunate candidates tried without any success to rearrange  $f(x) = 0$ . Most however were happy to find  $f'(x)$ , set it equal to 0 and then rearrange to find the desired result. As this was a given solution, the line  $0 f'(x) = 0$  was needed to be seen.

Part (c): Marks were generally lost due to degrees rather than a lack of accuracy. Otherwise this part resulted in 2 or 3 marks for most candidates.

Part (d): The majority of candidates gained the M mark for the correct interval (although an upper limit of 1.90784 was seen occasionally which normally would be acceptable, was not acceptable for this root). A large number lost the last two marks for substituting into the incorrect expression. A few candidates tried repeated iteration which was not the question. Only a small number completed the last part correctly however, with most substituting into  $f(x)$  or the iteration formula rather than  $f'(x)$ . Many who used  $f(x)$  made a comment statement there was a change in sign when there wasn't, and some even commented that they should have found a change in sign.

### Question 8

In part (a) most candidates knew that the quotient rule should be used. It would again be wise to quote this formula. Only high achieving candidates produced full solutions to part (a), with common mistakes including differentiating  $(3 + \sin 2x)$  to give either  $(2 \cos x)$  or  $(\cos 2x)$ , and similarly with  $(2 + \cos 2x)$ . Predictably, the final accuracy mark was frequently lost through



candidates not being explicit enough in their working to demonstrate the given result – jumping from  $(2 \cos^2(2x) + 2 \sin^2 2x)$  to 2 was common. This was a given solution and hence there was an expectation that the result should be shown. This could be achieved by writing  $(2 \cos^2 2x + 2 \sin^2 2x)$  as  $2(\cos^2 2x + \sin^2 2x) = 2 \times 1 = 2$ .

Some candidates struggled in part (b) to give a correct value for either  $y$  or  $m$ . A very common incorrect result was obtained by using the calculator set in degrees to work out these values. Occasionally, a “perpendicular method” was used to replace a gradient of  $-2$  with  $0.5$ . Most candidates arriving at an answer understood that exact answers for  $a$  and  $b$  were required.

## Statistics for C3 Practice Paper Silver Level S3

Qu	Max score	Modal score	Mean %	Mean score for students achieving grade:							
				ALL	A*	A	B	C	D	E	U
1	10		79	7.90		9.61	8.96	8.07	7.08	6.06	3.01
2	9		81	7.27	8.89	8.40	7.80	7.05	6.11	5.42	3.88
3	8		84	6.68		7.32	6.75	5.96	5.65	4.67	3.39
4	10		74	7.35	9.23	8.18	7.34	6.17	5.35	3.30	1.27
5	8		65	5.19	7.21	6.37	5.46	4.56	3.64	2.59	1.47
6	10		67	6.72		8.37	6.89	5.89	4.85	3.75	2.30
7	12		72	8.58	11.48	10.31	9.14	8.12	6.66	5.44	2.78
8	8		66	5.24	7.44	6.98	6.08	5.20	3.94	2.72	1.13
	<b>75</b>		<b>73</b>	<b>54.93</b>		<b>65.54</b>	<b>58.42</b>	<b>51.02</b>	<b>43.28</b>	<b>33.95</b>	<b>19.23</b>