Exercise 9E

Integrate with respect to x:

1 $(4x+5)^{\frac{1}{2}}$ 2 $\dfrac{1}{4x+5}$ 3 $\left(1-\dfrac{1}{x}\right)^2$

4 $\cos x \sin x$ 5 $\tan 3x$ 6 $x \sin 3x$

7 $\dfrac{1+x}{x^{\frac{1}{2}}}$ 8 $\dfrac{x}{1+x}$ 9 $\sin x \cos^4 x$

10 $3 \ln x$ 11 $\dfrac{x+2}{x(x-1)}$ 12 $\dfrac{\sec^2 x}{(1+\tan x)^3}$

13 $\sin^2 2x$ 14 $\dfrac{x^2}{x-2}$ 15 $(\sin x + 2\cos x)^2$

16 $x^2 e^{\frac{1}{2}x}$ 17 $\dfrac{1}{x^2-4}$ 18 $\dfrac{x}{9x^2+1}$

19 $(1-x^{-2})^2$ 20 $(2-3x)^{-2}$ 21 $(4-5x)^{-1}$

22 $\cot 3x$ 23 $\operatorname{cosec} 2x \cot 2x$ 24 $\cot^2 3x$

25 $x \cos 5x$ 26 $\dfrac{x}{(x-1)^{\frac{1}{2}}}$ 27 $x^2 e^{-x}$

28 $\cos 2x \sin x$ 29 $\sin 2x \cos x$ 30 $\tan 2x \sec 2x$

31 $\dfrac{(x+1)^2}{x^2+1}$ 32 $\dfrac{2}{(x-2)(x-4)}$ 33 $\dfrac{1}{x^2(x-1)}$

34 $\operatorname{cosec}^2 2x + 1$ 35 $\dfrac{x+4}{x-4}$ 36 $\dfrac{1}{x(x^2-1)}$

37 $\dfrac{x^2}{x^3+1}$ 38 $(e^x + x)^2$ 39 $x^3 \ln x$

40 $x^3 e^{x^2}$

41 Use the identity $\cos^2 x + \sin^2 x \equiv 1$ and the substitution $\cos x = u$ to find $\int \sin^3 x \, dx$.

42 Find $\int \cos^3 x \, dx$ and $\int \sin^5 x \, dx$.

43 Use the identity $\sec^2 x \equiv \tan^2 x + 1$ and the substitution $\tan x = u$ to find $\int \tan^4 x \, dx$.

44 Find (a) $\int \sec^4 x \, dx$ (b) $\int \cot^4 x \, dx$.

14 $\dfrac{1}{2(3-2x)}$ 15 $\frac{1}{2}e^{2x} - 2x - \frac{1}{2}e^{-2x}$

16 $-2\cot\frac{1}{2}x$ 17 $\frac{1}{3}\sec 3x$

18 $-\frac{1}{2}\operatorname{cosec} 2x$ 19 $\tan x - \frac{1}{3}x^3$

20 $x^2 - \frac{1}{2}\cos 2x$ 21 $\frac{1}{2}$ 22 $1\frac{1}{2}$ 23 0

24 1 25 $1 + \frac{1}{8}\pi^2$ 26 $2 - 2\sqrt{3}$

27 $\frac{1}{3}\ln\frac{5}{2}$ 28 10 29 $\frac{1}{2}(e - e^{-3})$ 30 2

Exercise 9B

[The constant of integration is omitted in indefinite integration.]

1 $\frac{1}{2}x - \frac{1}{4}\sin 2x$ 2 $-\cot x - x$

3 $\frac{1}{2}\tan 2x - x$ 4 $\dfrac{3x}{2} + 2\sin x + \frac{1}{4}\sin 2x$

5 $3x + 4\cos x - \sin 2x$

6 $\dfrac{5x}{2} + \frac{1}{4}\sin 2x + \tan x$ 7 $\frac{1}{4}\ln\left|\dfrac{x-2}{x+2}\right|$

8 $\ln|x-3| - 2\ln|x-2|$

9 $\ln|x-1| - 2\ln|2x+1|$

10 $\frac{1}{2}\ln|2x+1| - \frac{1}{3}\ln|3x+1|$

11 $\ln\left|\dfrac{2x+1}{3x+1}\right|$ 12 $\ln\left|\dfrac{3+2x}{3-2x}\right|$

13 $A = 1, B = \frac{1}{2}, C = -\frac{1}{2}$;

$x + \frac{1}{2}\ln\left|\dfrac{x-1}{x+1}\right|$

14 $\frac{\pi}{3} + \frac{1}{4}$ 15 0

16 $\frac{\sqrt{3}}{4} + \frac{1}{3}$ 17 (a) $\frac{\pi}{2} - 1$ (b) $\frac{\pi}{2} + 1$

Exercise 9C

[The constant of integration is omitted in indefinite integration]

1 $\frac{1}{4}\sin^4 x$ 2 $\frac{1}{3}\tan^3 x$ 3 $\frac{1}{8}(x^2+1)^4$

4 $\frac{1}{6}(x^4-1)^{\frac{3}{2}}$ 5 $(x^2-1)^{\frac{1}{2}}$ 6 $\frac{1}{2}\sec^2 x$

7 $\frac{1}{2}e^{x^2}$ 8 $\frac{1}{3}(\ln|x|)^3$

9 $x + 1 - 2\ln|x+1| - \dfrac{1}{x+1}$

10 $\frac{2}{3}(x-2)(x+1)^{\frac{1}{2}}$ 11 $6\frac{2}{3}$ 13 $-\frac{1}{12}$

14 (a) $e - 1$ (b) $\frac{1}{4}\sqrt{3}$ (c) $\ln\frac{5}{4}$

15 (a) $\frac{1}{2}\ln 2$ (b) $\frac{1}{2}\ln 2$

Exercise 9D

[The constant of integration is omitted in indefinite integration.]

1 $-e^{-x}(x+1)$ 2 $\frac{1}{3}xe^{3x} - \frac{1}{9}e^{3x}$

3 $-x\cos x + \sin x$

4 $\dfrac{x^2}{2}\ln|x| - \dfrac{x^2}{4}$

5 $x\ln|x-1| - x - \ln|x-1|$

6 $\frac{1}{3}x\sin 3x + \frac{1}{9}\cos 3x$

7 $\dfrac{(5x+1)(x-1))^5}{30}$ 8 $\frac{2}{15}(3x+2)(x-1)^{\frac{3}{2}}$

9 $e^x(x^2 - 2x + 2)$

10 $x^2\sin x + 2x\cos x - 2\sin x$

11 $-e^{-x}(x^2 + 2x + 2)$

12 $\dfrac{x^4}{16}(4\ln x - 1)$ 13 π

14 $\dfrac{\pi}{\sqrt{2}} + \dfrac{4}{\sqrt{2}} - 4$ 15 $\frac{2}{9}e^3 + \frac{1}{9}$ 16 $-\frac{1}{20}$

17 8.4 18 $\frac{1}{9}(1 - 4e^{-3})$ 19 $e - 2$

20 $\frac{1}{2}(e^{\frac{\pi}{2}} + 1)$

Exercise 9E

[The constant of integration is omitted in indefinite integration.]

1 $\frac{1}{6}(4x-5)^{\frac{3}{2}}$ 3 $\frac{1}{4}\ln|4x+5|$

3 $x - 2\ln|x| - \dfrac{1}{x}$ 4 $\frac{1}{2}\sin^2 x$

5 $\frac{1}{3}\ln|\sec 3x|$ 6 $-\frac{1}{3}x\cos 3x + \frac{1}{9}\sin 3x$

7 $2x^{\frac{1}{2}} + \frac{2}{3}x^{\frac{3}{2}}$ 8 $x - \ln|x+1|$

9 $-\frac{1}{5}\cos^5 x$ **10** $3x\ln x - 3x$

11 $3\ln|x-1| - 2\ln|x|$

12 $-\frac{1}{2}(1+\tan x)^{-2}$ **13** $\frac{1}{2}x - \frac{1}{8}\sin 4x$

14 $\frac{1}{2}x^2 + 2x + 4\ln|x-2|$

15 $\frac{5}{2}x + \frac{3}{4}\sin 2x - \cos 2x$

16 $2e^{\frac{1}{2}x}(x^2 - 4x + 8)$ **17** $\frac{1}{4}\ln\left|\frac{x-2}{x+2}\right|$

18 $\frac{1}{18}\ln|9x^2+1|$ **19** $x + 2x^{-1} - \frac{1}{3}x^{-3}$

20 $\frac{1}{3}(2-3x)^{-1}$ **21** $-\frac{1}{5}\ln|4-5x|$

22 $\frac{1}{3}\ln|\sin 3x|$ **23** $-\frac{1}{2}\operatorname{cosec} 2x$

24 $-\frac{1}{3}\cot 3x - x$ **25** $\frac{1}{5}x\sin 5x + \frac{1}{25}\cos 5x$

26 $\frac{2}{3}(x+2)\sqrt{(x-1)}$

27 $-e^{-x}(x^2+2x+2)$

28 $-\frac{2}{3}\cos^3 x + \cos x$ **29** $-\frac{2}{3}\cos^3 x$

30 $\frac{1}{2}\sec 2x$ **31** $x + \ln|1+x^2|$

32 $\ln\left|\frac{x-4}{x-2}\right|$ **33** $\frac{1}{x} + \ln\left|\frac{x-1}{x}\right|$

34 $x - \frac{1}{2}\cot 2x$ **35** $x + 8\ln|x-4|$

36 $\frac{1}{2}\ln\left|\frac{x^2-1}{x^2}\right|$ **37** $\frac{1}{3}\ln|x^3+1|$

38 $\frac{1}{2}e^{2x} + \frac{1}{3}x^3 + 2xe^x - 2e^x$

39 $\frac{1}{16}x^4(4\ln x - 1)$ **40** $\frac{1}{2}e^{x^2}(x^2-1)$

41 $\frac{1}{3}\cos^3 x - \cos x$

42 $\sin x - \frac{1}{3}\sin^3 x, -\cos x + \frac{2}{3}\cos^3 x - \frac{1}{5}\cos^5 x$

43 $\frac{1}{3}\tan^3 x - \tan x + x$

44 $\frac{1}{3}\tan^3 x + \tan x, x + \cot x - \frac{1}{3}\cot^3 x$

45 (a) $-\frac{1}{10}\cos 10x - \frac{1}{2}\cos 2x$

(b) $-\frac{1}{3}\cos\frac{3x}{2} - \cos\frac{x}{2}$

46 $\frac{5}{2}\ln 3 - \ln 2$

48 $\frac{1}{3}\left[8 - \frac{8}{3\sqrt{3}}\right]$ **49** $\ln 2 + \frac{1}{2}\ln 13 - \frac{1}{2}\ln 20$

50 $\frac{1}{2}\ln\frac{5}{2}$

Exercise 9F

1 $\frac{\sqrt{3}-1}{2}$ **2** 1 **3** $2e^3$

4 $5\ln 5 - 2\ln 2 - 3$ **5** $\frac{\pi}{6}$ **6** $2 - \frac{\pi}{4}$

7 0.02 **8** $\frac{1}{2}(e^4 - e)$ **9** $\ln\frac{4}{3}$ **10** $\frac{26}{27}\sqrt{3}$

11 8π **12** 168π **13** $\frac{1}{2}\pi^2$ **14** $\pi\ln\frac{5}{2}$

15 $\frac{\pi}{4}e^2(3e^2-1)$ **16** $347\frac{1}{15}\pi$

17 $\frac{\pi}{4}(4-\pi)$ **18** $\pi[3(\ln 3)^2 - 6\ln 3 + 4]$

19 $\frac{\pi}{4}[15 + 8\ln 4]$ **20** $\frac{64}{15}\pi$

24 $60, \frac{16\,256\pi}{7}$

25 (a) $\sqrt{2}-1$ (b) $\frac{\pi}{4}(\pi-2)$

26 (a) $12\frac{2}{3}$ (b) $32\frac{1}{2}\pi$

27 (a) 27 (b) $\frac{1773\pi}{5}$

28 $16\ln\frac{16}{3}, \frac{208}{3}\pi$ **29** (a) $\frac{3\pi}{2}$ (b) 12π

Exercise 9G

1 $y = \frac{1}{2}e^{2x-1} + C$ **2** $2x + e^{1-2y} = C$

3 $4y = 2x + \sin 2x + C$ **4** $\tan y = x + C$

5 $2\ln y = x^2 + C$ **6** $e^{-y} + e^x = C$

7 $\sin y = x\ln x - x + C$

8 $y + 2 = C(x+1)$

9 $\ln y = \ln x + \frac{x^2}{2} + C$

10 $y^2 = 2\operatorname{cosec} x + C$ **11** $6y = 2x^3 + 3x^2$

12 $3y = \sin^3 x - 1$ **13** $\ln(3y+1) = 3x - 3$

14 $2e^{-y} + x^2 - 3 = 0$ **15** $2\sin y = \sec x$

16 $2\tan y = x - \sin x\cos x$

17 $\sin^2 y = \frac{11}{4} - \frac{2}{x}$

18 $y^2 = 2\tan x(\tan x + 1) + 5$

19 $y\cos y - \sin y = \frac{\pi}{2} - x\sin x - \cos x$

20 $\frac{y-1}{y+1} = \frac{2\sin^2 x}{3}$