# C4 SERIES — Worksheet C

**1** **a** Expand $(1 - 4x)^{\frac{1}{2}}$ in ascending powers of $x$ up to and including the term in $x^3$ and state the set of values of $x$ for which the expansion is valid. **(4)**

**b** By substituting $x = 0.01$ in your expansion, find the value of $\sqrt{6}$ to 6 significant figures. **(3)**

**2** $$f(x) \equiv \frac{4}{1+2x-3x^2}.$$

**a** Express f($x$) in partial fractions. **(3)**

**b** Hence, or otherwise, find the series expansion of f($x$) in ascending powers of $x$ up to and including the term in $x^3$ and state the set of values of $x$ for which the expansion is valid. **(5)**

**3** **a** Expand $(2 - x)^{-2}$, $|x| < 2$, in ascending powers of $x$ up to and including the term in $x^3$. **(4)**

**b** Hence, find the coefficient of $x^3$ in the series expansion of $\dfrac{3-x}{(2-x)^2}$. **(2)**

**4** $$f(x) \equiv \frac{4}{\sqrt{1+\frac{2}{3}x}}, \quad -\tfrac{3}{2} < x < \tfrac{3}{2}.$$

**a** Show that $f(\frac{1}{10}) = \sqrt{15}$. **(2)**

**b** Expand f($x$) in ascending powers of $x$ up to and including the term in $x^2$. **(3)**

**c** Use your expansion to obtain an approximation for $\sqrt{15}$, giving your answer as an exact, simplified fraction. **(2)**

**d** Show that $3\frac{55}{63}$ is a more accurate approximation for $\sqrt{15}$. **(2)**

**5** **a** Expand $(1 - x)^{\frac{1}{3}}$, $|x| < 1$, in ascending powers of $x$ up to and including the term in $x^2$. **(3)**

**b** By substituting $x = 10^{-3}$ in your expansion, find the cube root of 37 correct to 9 significant figures. **(3)**

**6** The series expansion of $(1 + 5x)^{\frac{3}{5}}$, in ascending powers of $x$ up to and including the term in $x^3$, is

$$1 + 3x + px^2 + qx^3, \quad |x| < \tfrac{1}{5}.$$

**a** Find the values of the constants $p$ and $q$. **(4)**

**b** Use the expansion with a suitable value of $x$ to find an approximate value for $(1.1)^{\frac{3}{5}}$. **(2)**

**c** Obtain the value of $(1.1)^{\frac{3}{5}}$ from your calculator and hence find the percentage error in your answer to part **b**. **(2)**

**7** **a** Find the values of $A$, $B$ and $C$ such that

$$\frac{8-6x^2}{(1+x)(2+x)^2} \equiv \frac{A}{1+x} + \frac{B}{2+x} + \frac{C}{(2+x)^2}.$$ **(4)**

**b** Hence find the series expansion of $\dfrac{8-6x^2}{(1+x)(2+x)^2}$, $|x| < 1$, in ascending powers of $x$ up to and including the term in $x^3$, simplifying each coefficient. **(7)**

**8** **a** Expand $(1 - 2x)^{\frac{1}{2}}$, $|x| < \frac{1}{2}$, in ascending powers of $x$ up to and including the term in $x^2$. **(3)**

**b** By substituting $x = 0.0008$ in your expansion, find the square root of 39 correct to 7 significant figures. **(4)**

**9** **a** Find the series expansion of $(1 + 8x)^{\frac{1}{3}}$, $|x| < \frac{1}{8}$, in ascending powers of $x$ up to and including the term in $x^2$, simplifying each term. **(3)**

**b** Find the exact fraction $k$ such that

$$\sqrt[3]{5} = k\sqrt[3]{1.08}$$ **(2)**

**c** Hence, use your answer to part **a** together with a suitable value of $x$ to obtain an estimate for $\sqrt[3]{5}$, giving your answer to 4 significant figures. **(3)**

**10**
$$f(x) \equiv \frac{6x}{x^2 - 4x + 3}, \quad |x| < 1.$$

**a** Express f($x$) in partial fractions. **(3)**

**b** Show that for small values of $x$,

$$f(x) \approx 2x + \tfrac{8}{3}x^2 + \tfrac{26}{9}x^3.$$ **(5)**

**11** **a** Find the binomial expansion of $(4 + x)^{\frac{1}{2}}$ in ascending powers of $x$ up to and including the term in $x^2$ and state the set of values of $x$ for which the expansion is valid. **(4)**

**b** By substituting $x = \frac{1}{20}$ in your expansion, find an estimate for $\sqrt{5}$, giving your answer to 9 significant figures. **(3)**

**c** Obtain the value of $\sqrt{5}$ from your calculator and hence comment on the accuracy of the estimate found in part **b**. **(2)**

**12** **a** Expand $(1 + 2x)^{-\frac{1}{2}}$, $|x| < \frac{1}{2}$, in ascending powers of $x$ up to and including the term in $x^3$. **(4)**

**b** Hence, show that for small values of $x$,

$$\frac{2 - 5x}{\sqrt{1 + 2x}} \approx 2 - 7x + 8x^2 - \tfrac{25}{2}x^3.$$ **(3)**

**c** Solve the equation

$$\frac{2 - 5x}{\sqrt{1 + 2x}} = \sqrt{3}.$$ **(3)**

**d** Use your answers to parts **b** and **c** to find an approximate value for $\sqrt{3}$. **(2)**

**13** **a** Expand $(1 + x)^{-1}$, $|x| < 1$, in ascending powers of $x$ up to and including the term in $x^3$. **(2)**

**b** Hence, write down the first four terms in the expansion in ascending powers of $x$ of $(1 + bx)^{-1}$, where $b$ is a constant, for $|bx| < 1$. **(1)**

Given that in the series expansion of

$$\frac{1 + ax}{1 + bx}, \quad |bx| < 1,$$

the coefficient of $x$ is –4 and the coefficient of $x^2$ is 12,

**c** find the values of the constants $a$ and $b$, **(5)**

**d** find the coefficient of $x^3$ in the expansion. **(2)**