1 **a** $f(x) = x[x^3 + 3x^2(2) + 3x(2)^2 + 2^3]$
$= x^4 + 6x^3 + 12x^2 + 8x$
$f'(x) = 4x^3 + 18x^2 + 24x + 8$

b $f'(x) = 1 \times (x + 2)^3 + x \times 3(x + 2)^2$
$= (x + 2)^2[(x + 2) + 3x]$
$= 2(2x + 1)(x + 2)^2$

2 **a** $= 1 \times e^x + x \times e^x$
$= e^x(1 + x)$

b $= 1 \times (x + 1)^5 + x \times 5(x + 1)^4$
$= (x + 1)^4[(x + 1) + 5x]$
$= (6x + 1)(x + 1)^4$

c $= 1 \times \ln x + x \times \frac{1}{x}$
$= \ln x + 1$

d $= 2x \times (x - 1)^3 + x^2 \times 3(x - 1)^2$
$= x(x - 1)^2[2(x - 1) + 3x]$
$= x(5x - 2)(x - 1)^2$

e $= 3x^2 \times \ln 2x + x^3 \times \frac{1}{x}$
$= x^2(3 \ln 2x + 1)$

f $= 2x \times e^{-x} + x^2 \times (-e^{-x})$
$= xe^{-x}(2 - x)$

g $= 8x^3 \times (5 + x)^3 + 2x^4 \times 3(5 + x)^2$
$= 2x^3(5 + x)^2[4(5 + x) + 3x]$
$= 2x^3(20 + 7x)(5 + x)^2$

h $= 2x \times (x - 3)^4 + x^2 \times 4(x - 3)^3$
$= 2x(x - 3)^3[(x - 3) + 2x]$
$= 6x(x - 1)(x - 3)^3$

3 **a** $= 1 \times (2x - 1)^3 + x \times 3(2x - 1)^2 \times 2$
$= (2x - 1)^2[(2x - 1) + 6x]$
$= (8x - 1)(2x - 1)^2$

b $= 12x^3 \times e^{2x + 3} + 3x^4 \times e^{2x + 3} \times 2$
$= 6x^3e^{2x + 3}(2 + x)$

c $= 1 \times \sqrt{x-1} + x \times \frac{1}{2}(x-1)^{-\frac{1}{2}}$
$= \frac{1}{2}(x-1)^{-\frac{1}{2}}[2(x - 1) + x]$
$= \frac{3x-2}{2\sqrt{x-1}}$

d $= 2x \times \ln(x + 6) + x^2 \times \frac{1}{x+6}$
$= 2x \ln(x + 6) + \frac{x^2}{x+6}$

e $= 1 \times (1 - 5x)^4 + x \times 4(1 - 5x)^3 \times (-5)$
$= (1 - 5x)^3[(1 - 5x) - 20x]$
$= (1 - 25x)(1 - 5x)^3$

f $= 1 \times (x - 3)^3 + (x + 2) \times 3(x - 3)^2$
$= (x - 3)^2[(x - 3) + 3(x + 2)]$
$= (4x + 3)(x - 3)^2$

g $= \frac{4}{3}x^{\frac{1}{3}} \times e^{3x} + x^{\frac{4}{3}} \times 3e^{3x}$
$= \frac{1}{3}x^{\frac{1}{3}}e^{3x}(4 + 9x)$

h $= 1 \times \ln(x^2 - 1) + (x + 1) \times \frac{1}{x^2-1} \times 2x$
$= \ln(x^2 - 1) + \frac{2x(x+1)}{(x+1)(x-1)}$
$= \ln(x^2 - 1) + \frac{2x}{x-1}$

i $= 2x \times \sqrt{3x+1} + x^2 \times \frac{1}{2}(3x+1)^{-\frac{1}{2}} \times 3$
$= \frac{1}{2}x(3x+1)^{-\frac{1}{2}}[4(3x + 1) + 3x]$
$= \frac{x(15x+4)}{2\sqrt{3x+1}}$

4 **a** $f'(x) = 4 \times e^{3x} + 4x \times 3e^{3x}$
$= 4e^{3x}(1 + 3x)$
$f'(0) = 4 \times 1 \times 1 = 4$

b $f'(x) = 2 \times (x^2 + 2)^3 + 2x \times 3(x^2 + 2)^2 \times 2x$
$= 2(x^2 + 2)^2[(x^2 + 2) + 6x^2]$
$= 2(7x^2 + 2)(x^2 + 2)^2$
$f'(-1) = 2 \times 9 \times 9 = 162$

c $f'(x) = 5 \times \ln 3x + (5x - 4) \times \frac{1}{x}$
$= 5 \ln 3x + 5 - \frac{4}{x}$
$f'(\frac{1}{3}) = 0 + 5 - 12 = -7$

d $f'(x) = \frac{1}{2}x^{-\frac{1}{2}} \times (1 - 2x)^3 + x^{\frac{1}{2}} \times 3(1 - 2x)^2 \times (-2)$
$= \frac{1}{2}x^{-\frac{1}{2}}(1 - 2x)^2[(1 - 2x) - 12x]$
$= \frac{1}{2}x^{-\frac{1}{2}}(1 - 14x)(1 - 2x)^2$
$f'(\frac{1}{4}) = \frac{1}{2} \times 2 \times (-\frac{5}{2}) \times \frac{1}{4} = -\frac{5}{8}$

5 **a** $\frac{dy}{dx} = 1 \times e^{2x} + x \times 2e^{2x}$
SP: $e^{2x}(1 + 2x) = 0$
$x = -\frac{1}{2}$
$\therefore (-\frac{1}{2}, -\frac{1}{2}e^{-1})$

b $\frac{dy}{dx} = 1 \times (x - 4)^3 + x \times 3(x - 4)^2$
$= (x - 4)^2[(x - 4) + 3x]$
SP: $4(x - 1)(x - 4)^2 = 0$
$x = 1, 4$
$\therefore (1, -27), (4, 0)$

c $\frac{dy}{dx} = 2x \times (2x - 3)^4 + x^2 \times 4(2x - 3)^3 \times 2$
$= 2x(2x - 3)^3[(2x - 3) + 4x]$
SP: $6x(2x - 1)(2x - 3)^3 = 0$
$x = 0, \frac{1}{2}, \frac{3}{2}$
$\therefore (0, 0), (\frac{1}{2}, 4), (\frac{3}{2}, 0)$

d $\frac{dy}{dx} = 1 \times \sqrt{x+12} + x \times \frac{1}{2}(x+12)^{-\frac{1}{2}}$
$= \frac{1}{2}(x+12)^{-\frac{1}{2}}[2(x + 12) + x]$
SP: $\frac{3(x+8)}{2\sqrt{x+12}} = 0$
$x = -8$
$\therefore (-8, -16)$

e $\frac{dy}{dx} = 2x \times e^{-4x} + x^2 \times (-4e^{-4x})$
SP: $2xe^{-4x}(1 - 2x) = 0$
$x = \frac{1}{2}$
$\therefore (\frac{1}{2}, 2 + \frac{1}{4}e^{-2})$

f $\frac{dy}{dx} = -3 \times (3 - x)^3 + (1 - 3x) \times 3(3 - x)^2 \times (-1)$
$= -3(3 - x)^2[(3 - x) + (1 - 3x)]$
SP: $-12(1 - x)(3 - x)^2 = 0$
$x = 1, 3$
$\therefore (1, -16), (3, 0)$

6 **a** $x = 1 \ \therefore y = 1$
$\frac{dy}{dx} = 1 \times (x - 2)^4 + x \times 4(x - 2)^3$
$= (x - 2)^3[(x - 2) + 4x]$
$= (5x - 2)(x - 2)^3$
grad $= -3$
$\therefore y - 1 = -3(x - 1)$
$[y = 4 - 3x]$

b $x = 1 \ \therefore y = 3e$
$\frac{dy}{dx} = 6x \times e^x + 3x^2 \times e^x$
$= 3xe^x(2 + x)$
grad $= 9e$
$\therefore y - 3e = 9e(x - 1)$
$[y = 3e(3x - 2)]$

c $x = \frac{1}{2} \ \therefore y = 0$
$\frac{dy}{dx} = 4 \times \ln 2x + (4x - 1) \times \frac{1}{x}$
$= 4 \ln 2x + 4 - \frac{1}{x}$
grad $= 2$
$\therefore y - 0 = 2(x - \frac{1}{2})$
$[y = 2x - 1]$

d $x = -2 \ \therefore y = 8$
$\frac{dy}{dx} = 2x \times \sqrt{x+6} + x^2 \times \frac{1}{2}(x+6)^{-\frac{1}{2}}$
$= \frac{1}{2}x(x+6)^{-\frac{1}{2}}[4(x + 6) + x] = \frac{x(5x+24)}{2\sqrt{x+6}}$
grad $= -7$
$\therefore y - 8 = -7(x + 2)$
$[y = -7x - 6]$

7 **a** $x = 1 \ \therefore \ y = 1$

$\frac{dy}{dx} = 2x \times (2-x)^3 + x^2 \times 3(2-x)^2 \times (-1)$

$= x(2-x)^2[2(2-x) - 3x]$

$= x(4-5x)(2-x)^2$

grad $= -1$

$\therefore$ grad of normal $= 1$

$\therefore \ y - 1 = 1(x-1)$

$x - y = 0$

b $x = 2 \ \therefore \ y = 0$

$\frac{dy}{dx} = 1 \times \ln(3x-5) + x \times \frac{1}{3x-5} \times 3$

$= \ln(3x-5) + \frac{3x}{3x-5}$

grad $= 6$

$\therefore$ grad of normal $= -\frac{1}{6}$

$\therefore \ y - 0 = -\frac{1}{6}(x-2)$

$x + 6y - 2 = 0$

c $x = 0 \ \therefore \ y = -1$

$\frac{dy}{dx} = 2x \times e^{3x} + (x^2-1) \times 3e^{3x}$

$= e^{3x}[2x + 3(x^2-1)]$

$= e^{3x}(3x^2 + 2x - 3)$

grad $= -3$

$\therefore$ grad of normal $= \frac{1}{3}$

$\therefore \ y + 1 = \frac{1}{3}(x-0)$

$x - 3y - 3 = 0$

d $x = 8 \ \therefore \ y = 16$

$\frac{dy}{dx} = 1 \times \sqrt{x-4} + x \times \frac{1}{2}(x-4)^{-\frac{1}{2}}$

$= \frac{1}{2}(x-4)^{-\frac{1}{2}}[2(x-4) + x]$

$= \frac{3x-8}{2\sqrt{x-4}}$

grad $= 4$

$\therefore$ grad of normal $= -\frac{1}{4}$

$\therefore \ y - 16 = -\frac{1}{4}(x-8)$

$x + 4y - 72 = 0$

8 **a** $x = 1 \ \therefore \ y = e$

$\frac{dy}{dx} = 1 \times e^{x^2} + x \times e^{x^2} \times 2x$

$= e^{x^2}(1 + 2x^2)$

grad $= 3e$

$\therefore \ y - e = 3e(x-1)$

$[\, y = e(3x-2) \,]$

b $x = 0 \ \Rightarrow \ y = -2e$

$y = 0 \ \Rightarrow \ x = \frac{2}{3}$

$\therefore$ area $= \frac{1}{2} \times 2e \times \frac{2}{3} = \frac{2}{3}e$