

1 $x = -1, y = 8$

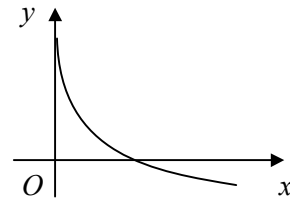
$$\frac{dy}{dx} = \frac{3}{2}(3-x)^{\frac{1}{2}} \times (-1) = -\frac{3}{2}(3-x)^{\frac{1}{2}}$$

grad = -3

$$\therefore y - 8 = -3(x + 1)$$

$$[y = 5 - 3x]$$

2 a



b $y = 0 \therefore x = \frac{1}{2}e^3$

$$\left(\frac{1}{2}e^3, 0\right)$$

c $x = 5 \therefore y = 3 - \ln 10$

$$\frac{dy}{dx} = -\frac{1}{x}, \text{ grad} = -\frac{1}{5}$$

$$\therefore y - (3 - \ln 10) = -\frac{1}{5}(x - 5)$$

$$[y = -\frac{1}{5}x + 4 - \ln 10]$$

d at A, $y = 0 \therefore x = 5(4 - \ln 10)$

at B, $x = 0 \therefore y = 4 - \ln 10$

$$\begin{aligned} \text{area} &= \frac{1}{2} \times 5(4 - \ln 10) \times (4 - \ln 10) \\ &= 7.20 \text{ (3sf)} \end{aligned}$$

3 a $= 4(3x - 1)^3 \times 3$

$$= 12(3x - 1)^3$$

b $= \frac{2x \times \sin 2x - x^2 \times 2 \cos 2x}{\sin^2 2x}$

$$= \frac{2x(\sin 2x - x \cos 2x)}{\sin^2 2x}$$

4 a $t = 3 \therefore \text{area} = 2e^{1.5} = 8.96 \text{ cm}^2 \text{ (3sf)}$

b $\frac{dA}{dt} = 2 \times 0.5e^{0.5t} = e^{0.5t}$

$$t = 3, \frac{dA}{dt} = e^{1.5} = 4.4817 \text{ cm}^2 \text{ yr}^{-1}$$

$$\therefore \text{rate per day} = 4.4817 \div 365 = 0.0123$$

area increasing at 0.0123 cm² per day (3sf)

c $65 = 2e^{0.5t}$

$$t = 2 \ln 32.5 = 6.96$$

$$\therefore 7 \text{ years}$$

d A increases exponentially and would become larger than the surface area of the boulder

5 a $\frac{dy}{dx} = \frac{a}{x} - 4$

SP: $\frac{a}{x} - 4 = 0$

$$x = \frac{1}{4}a$$

$$\therefore \left(\frac{1}{4}a, a \ln \frac{a}{4} - a\right)$$

b $x = 1 \therefore y = -4, \text{ grad} = a - 4$

$$\therefore y + 4 = (a - 4)(x - 1)$$

$$[y = (a - 4)x - a]$$

c $(3, 0) \therefore 0 = 3(a - 4) - a$
 $a = 6$

6 a $\frac{dy}{dx} = 2e^{2x} \sin x + e^{2x} \cos x$

$$= e^{2x}(2 \sin x + \cos x)$$

b $\frac{d^2y}{dx^2} = 2e^{2x}(2 \sin x + \cos x) + e^{2x}(2 \cos x - \sin x)$

$$= e^{2x}(3 \sin x + 4 \cos x)$$

$$\therefore \frac{d^2y}{dx^2} - 4 \frac{dy}{dx} + 5y$$

$$= e^{2x}(3 \sin x + 4 \cos x)$$

$$- 4e^{2x}(2 \sin x + \cos x) + 5e^{2x} \sin x$$

$$= e^{2x}(3 \sin x + 4 \cos x - 8 \sin x - 4 \cos x + 5 \sin x)$$

$$= 0$$

$$\begin{aligned}
 7 \quad \mathbf{a} \quad \frac{dx}{dy} &= 2 \tan y \sec^2 y \\
 &= 2 \tan y (\tan^2 y + 1) \\
 &= 2\sqrt{x} (x + 1)
 \end{aligned}$$

$$\frac{dy}{dx} = 1 \div \frac{dx}{dy} = \frac{1}{2\sqrt{x}(x+1)}$$

$$\begin{aligned}
 \mathbf{b} \quad y &= \frac{\pi}{4} \quad \therefore x = 1 \\
 \text{grad} &= \frac{1}{4} \\
 \therefore \text{grad of normal} &= -4 \\
 \therefore y - \frac{\pi}{4} &= -4(x - 1) \\
 [16x + 4y - \pi - 16 &= 0]
 \end{aligned}$$

$$\begin{aligned}
 9 \quad \mathbf{a} &= e^x \times (x - 1)^2 + e^x \times 2(x - 1) \times 1 \\
 &= e^x(x^2 - 2x + 1 + 2x - 2)
 \end{aligned}$$

$$\begin{aligned}
 &= e^x(x^2 - 1) \\
 \mathbf{b} &= e^x \times (x^2 - 1) + e^x \times 2x \\
 &= e^x(x^2 + 2x - 1)
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{c} \quad \text{SP: } e^x(x^2 - 1) &= 0 \\
 x &= \pm 1 \\
 \therefore (-1, 4e^{-1}), (1, 0) \\
 (-1, 4e^{-1}), \frac{d^2y}{dx^2} &= -2e^{-1} \quad \therefore \text{maximum} \\
 (1, 0), \frac{d^2y}{dx^2} &= 2e \quad \therefore \text{minimum}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{d} \quad x = 2 \quad \therefore y &= e^2 \\
 \text{grad} &= 3e^2 \\
 \therefore y - e^2 &= 3e^2(x - 2) \\
 y &= 3e^2x - 5e^2 \\
 y &= e^2(3x - 5)
 \end{aligned}$$

$$\begin{aligned}
 11 \quad \mathbf{a} \quad f(x) &= \frac{6x - 2(x+2)}{(x-1)(x+2)} = \frac{4x - 4}{(x-1)(x+2)} \\
 &= \frac{4(x-1)}{(x-1)(x+2)} = \frac{4}{x+2}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{b} \quad x = 2 \quad \therefore y &= 1 \\
 f'(x) &= -4(x+2)^{-2} \\
 \text{grad} &= -\frac{1}{4} \\
 \therefore y - 1 &= -\frac{1}{4}(x - 2) \\
 4y - 4 &= -x + 2 \\
 x + 4y &= 6
 \end{aligned}$$

$$\begin{aligned}
 8 \quad \mathbf{a} \quad \frac{dy}{dx} &= \frac{1 \times \sqrt{x-2} - (x+2) \times \frac{1}{2}(x-2)^{-\frac{1}{2}}}{x-2} \\
 &= \frac{2(x-2) - (x+2)}{2(x-2)^{\frac{3}{2}}} \\
 &= \frac{x-6}{2(x-2)^{\frac{3}{2}}}
 \end{aligned}$$

$$\mathbf{b} \quad \text{SP: } \frac{x-6}{2(x-2)^{\frac{3}{2}}} = 0$$

$$x = 6 \quad \therefore (6, 4)$$

$$\mathbf{c} \quad x = 3 \quad \therefore y = 5, \text{ grad} = -\frac{3}{2}$$

$$\therefore \text{grad of normal} = \frac{2}{3}$$

$$\therefore y - 5 = \frac{2}{3}(x - 3)$$

$$3y - 15 = 2x - 6$$

$$2x - 3y + 9 = 0$$

$$10 \quad \mathbf{a} \quad \frac{dy}{dx} = x - \frac{3}{x}$$

$$\text{SP: } x - \frac{3}{x} = 0$$

$$x^2 = 3$$

$$x > 0 \quad \therefore x = \sqrt{3}$$

$$\mathbf{b} \quad \frac{d^2y}{dx^2} = 1 + 3x^{-2}$$

$$x = \sqrt{3}, \frac{d^2y}{dx^2} = 2$$

$$\therefore \text{minimum}$$

$$\mathbf{c} \quad y = \frac{1}{2}(\sqrt{3})^2 - 3 \ln \sqrt{3}$$

$$= \frac{3}{2} - 3 \ln 3^{\frac{1}{2}}$$

$$= \frac{3}{2} - \frac{3}{2} \ln 3$$

$$= \frac{3}{2}(1 - \ln 3)$$

$$\mathbf{d} \quad x = 1 \quad \therefore y = \frac{1}{2}$$

$$\text{grad} = -2$$

$$\therefore y - \frac{1}{2} = -2(x - 1)$$

$$4x + 2y - 5 = 0$$