1   Find an equation for the tangent to the curve with equation

$$y = (3 - x)^{\frac{3}{2}}$$

at the point on the curve with x-coordinate -1. **(4)**

2   **a** Sketch the curve with equation $y = 3 - \ln 2x$. **(2)**

   b Find the exact coordinates of the point where the curve crosses the x-axis. **(2)**

   c Find an equation for the tangent to the curve at the point on the curve where $x = 5$. **(4)**

   This tangent cuts the x-axis at A and the y-axis at B.

   d Show that the area of triangle OAB, where O is the origin, is approximately 7.20 **(3)**

3   Differentiate with respect to x

   a $(3x - 1)^4$, **(2)**

   b $\dfrac{x^2}{\sin 2x}$. **(3)**

4   The area of the surface of a boulder covered by lichen, A cm^2, at time t years after initial observation, is modelled by the formula

$$A = 2e^{0.5t}.$$
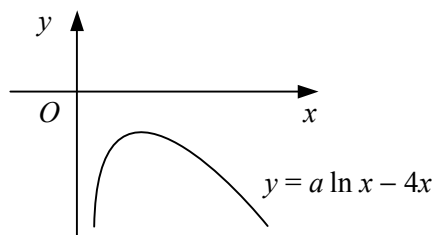
   Using this model,

   a find the area of lichen on the boulder after three years, **(2)**

   b find the rate at which the area of lichen is increasing per day after three years, **(4)**

   c find, to the nearest year, how long it takes until the area of lichen is 65 cm^2. **(2)**

   d Explain why the model cannot be valid for large values of t. **(1)**

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   The diagram shows the curve with equation $y = a \ln x - 4x$, where a is a positive constant.

   Find, in terms of a,

   a the coordinates of the stationary point on the curve, **(4)**

   b an equation for the tangent to the curve at the point where $x = 1$. **(3)**

   Given that this tangent meets the x-axis at the point $(3, 0)$,

   c show that $a = 6$. **(2)**

6   Given that $y = e^{2x} \sin x$,

   a find $\dfrac{dy}{dx}$, **(2)**
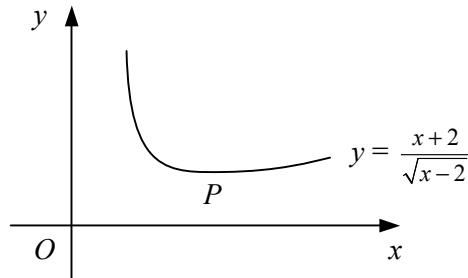
   b show that $\dfrac{d^2y}{dx^2} - 4\dfrac{dy}{dx} + 5y = 0$. **(3)**

7 A curve has the equation $x = \tan^2 y$.

   a Show that $\dfrac{dy}{dx} = \dfrac{1}{2\sqrt{x}(x+1)}$.   **(5)**

   b Find an equation for the normal to the curve at the point where $y = \frac{\pi}{4}$.   **(3)**

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The diagram shows the curve $y = \dfrac{x+2}{\sqrt{x-2}}$, $x > 2$, which has a minimum point at P.

   a Find and simplify an expression for $\dfrac{dy}{dx}$.   **(3)**

   b Find the coordinates of P.   **(2)**

The point Q on the curve has x-coordinate 3.

   c Show that the normal to the curve at Q has equation

     $2x - 3y + 9 = 0$.   **(3)**

9 A curve has the equation $y = e^x(x-1)^2$.

   a Find $\dfrac{dy}{dx}$.   **(3)**

   b Show that $\dfrac{d^2y}{dx^2} = e^x(x^2 + 2x - 1)$.   **(2)**

   c Find the exact coordinates of the turning points of the curve and determine their nature.   **(4)**

   d Show that the tangent to the curve at the point where $x = 2$ has the equation

     $y = e^2(3x - 5)$.   **(3)**

10 The curve with equation $y = \frac{1}{2}x^2 - 3\ln x$, $x > 0$, has a stationary point at A.

   a Find the exact x-coordinate of A.   **(3)**

   b Determine the nature of the stationary point.   **(2)**

   c Show that the y-coordinate of A is $\frac{3}{2}(1 - \ln 3)$.   **(2)**

   d Find an equation for the tangent to the curve at the point where $x = 1$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.   **(3)**

11 $f(x) = \dfrac{6x}{(x-1)(x+2)} - \dfrac{2}{x-1}$.

   a Show that $f(x) = \dfrac{4}{x+2}$.   **(5)**

   b Find an equation for the tangent to the curve $y = f(x)$ at the point with x-coordinate 2, giving your answer in the form $ax + by = c$, where a, b and c are integers.   **(4)**