

1  $f: x \rightarrow 4x - 3, x \in \mathbb{R}$      $g: x \rightarrow 2 - x, x \in \mathbb{R}$      $h: x \rightarrow x^2 + 5, x \in \mathbb{R}$

Evaluate

**a**  $gf(2)$                       **b**  $gh(1)$                       **c**  $fg(-3)$                       **d**  $hf(3)$   
**e**  $gg(5)$                       **f**  $ff(\frac{1}{2})$                       **g**  $hg(8)$                       **h**  $fh(1\frac{1}{2})$

2  $f: x \rightarrow 5x + 2, x \in \mathbb{R}$      $g: x \rightarrow \cos x, x \in \mathbb{R}$      $h: x \rightarrow \ln x, x \in \mathbb{R}, x > 0$

Evaluate, giving your answers to 3 significant figures

**a**  $fh(20)$                       **b**  $gh(3)$                       **c**  $fg(5)$                       **d**  $gg(-4)$   
**e**  $gf(1\frac{3}{4})$                       **f**  $hg(6.7)$                       **g**  $hh(50)$                       **h**  $hf(-0.3)$

3  $f: x \rightarrow 2x + 1, x \in \mathbb{R}$      $g: x \rightarrow 1 - 3x, x \in \mathbb{R}$      $h: x \rightarrow x^2 + 4, x \in \mathbb{R}$

Given the functions  $f$ ,  $g$  and  $h$ , express the following composite functions in a similar form.

**a**  $fg$                               **b**  $ff$                               **c**  $fh$                               **d**  $hf$   
**e**  $gh$                               **f**  $gg$                               **g**  $hg$                               **h**  $gf$

4  $f: x \rightarrow 4 - x, x \in \mathbb{R}$      $g: x \rightarrow e^x, x \in \mathbb{R}$      $h: x \rightarrow 2x^2 + 7, x \in \mathbb{R}$

Given the functions  $f$ ,  $g$  and  $h$ , express the following composite functions in a similar form.

**a**  $gf$                               **b**  $hg$                               **c**  $fh$                               **d**  $gg$   
**e**  $gh$                               **f**  $ff$                               **g**  $fg$                               **h**  $hf$

5  $f: x \rightarrow 5x - 3, x \in \mathbb{R}$      $g: x \rightarrow 3x^2 + 1, x \in \mathbb{R}$      $h: x \rightarrow \frac{1}{x-2}, x \in \mathbb{R}, x \neq 2$

Solve

**a**  $ff(x) = -8$                       **b**  $hf(x) = 2$                       **c**  $gf(x) = 28$                       **d**  $hg(x) = \frac{1}{2}$   
**e**  $fh(x) = 7$                       **f**  $fg(x) = 32$                       **g**  $gh(x) = 4$                       **h**  $hh(x) = -2$

6  $f: x \rightarrow \ln x, x \in \mathbb{R}, x > 0$      $g: x \rightarrow 3 + 2x, x \in \mathbb{R}$      $h: x \rightarrow e^x, x \in \mathbb{R}$

Solve, giving your answers to 2 decimal places,

**a**  $gh(x) = 9$                       **b**  $fg(x) = 3.6$                       **c**  $hg(x) = 4$                       **d**  $gf(x) = 10.4$

7 The functions  $f$  and  $g$  are defined by

$$f: x \rightarrow \frac{x+1}{5}, x \in \mathbb{R} \qquad g: x \rightarrow e^x, x \in \mathbb{R}$$

- a** State the range of  $g$ .  
**b** Solve  $fg(x) = 17$ .

8 The functions  $f$  and  $g$  are defined by

$$f(x) \equiv 4x - 9, x \in \mathbb{R} \qquad g(x) \equiv x^2, x \in \mathbb{R}$$

- a** Evaluate  $ff(3\frac{1}{4})$ .  
**b** Solve  $gf(x) = 25$ .  
**c** Sketch the graph of  $y = fg(x)$ , showing the coordinates of any points of intersection with the coordinate axes.

9  $f: x \rightarrow \tan x, x \in \mathbb{R}$      $g: x \rightarrow 4 + \ln x, x \in \mathbb{R}^+$      $h: x \rightarrow e^{2x-1}, x \in \mathbb{R}$

Evaluate

**a**  $gf(\frac{\pi}{4})$                       **b**  $hg(e^{-2})$                       **c**  $gh(-1)$                       **d**  $ff(1)$   
**e**  $hf(0.2)$                       **f**  $fg(7)$                       **g**  $hh(\frac{1}{4})$                       **h**  $fg(e^e)$

10  $f: x \rightarrow 3e^x + 2, x \in \mathbb{R}$      $g: x \rightarrow 4x + 1, x \in \mathbb{R}$      $h: x \rightarrow \frac{1}{x+1}, x \in \mathbb{R}, x \neq -1$

Express the following composite functions in a similar form, stating the domain in each case.

**a**  $fg$                       **b**  $gf$                       **c**  $hf$                       **d**  $gg$   
**e**  $hg$                       **f**  $gh$                       **g**  $hh$                       **h**  $ggg$

11  $f: x \rightarrow \sqrt{x+4}, x \in \mathbb{R}, x > -4$      $g: x \rightarrow e^{1+2x}, x \in \mathbb{R}$      $h: x \rightarrow \frac{x+1}{3}, x \in \mathbb{R}$

Solve

**a**  $fh(x) = 3$                       **b**  $fg(x) = 7$                       **c**  $gh(x) = 11$                       **d**  $hh(x) = \frac{2}{3}$   
**e**  $hg(x) = 1.2$                       **f**  $hf(x) = \frac{1}{2}$                       **g**  $ff(x) = 3$                       **h**  $ghh(x) = \frac{1}{2}$

12  $f(x) \equiv x^3, x \in \mathbb{R}$                        $g(x) \equiv x + 2, x \in \mathbb{R}$

Find the composition of the functions  $f$  and  $g$  that corresponds to the function  $h$ , where

**a**  $h(x) \equiv (x+2)^3, x \in \mathbb{R}$                       **b**  $h(x) \equiv x^3 + 2, x \in \mathbb{R}$                       **c**  $h(x) \equiv x + 4, x \in \mathbb{R}$   
**d**  $h(x) \equiv x^9, x \in \mathbb{R}$                       **e**  $h(x) \equiv x^9 + 2, x \in \mathbb{R}$                       **f**  $h(x) \equiv (x+2)^3 + 2, x \in \mathbb{R}$

13  $f(x) \equiv x - 4, x \in \mathbb{R}$                        $g(x) \equiv 3x^2, x \in \mathbb{R}$                        $h(x) \equiv \frac{1}{x}, x \in \mathbb{R}, x \neq 0$

Find the composition of the functions  $f, g$  and  $h$  that corresponds to the function  $j$ , where

**a**  $j(x) \equiv 3x^2 - 4, x \in \mathbb{R}$                       **b**  $j(x) \equiv \frac{1}{x-4}, x \in \mathbb{R}, x \neq 4$   
**c**  $j(x) \equiv \frac{3}{x^2}, x \in \mathbb{R}, x \neq 0$                       **d**  $j(x) \equiv 27x^4, x \in \mathbb{R}$   
**e**  $j(x) \equiv \frac{1}{3x^2} - 4, x \in \mathbb{R}, x \neq 0$                       **f**  $j(x) \equiv \frac{1}{3x^2 - 4}, x \in \mathbb{R}, x \neq \pm \frac{2}{\sqrt{3}}$

14 The functions  $f$  and  $g$  are defined by

$$f: x \rightarrow 5^x - 7, x \in \mathbb{R} \qquad g: x \rightarrow 2x + 3, x \in \mathbb{R}$$

- a** Find and simplify an expression for  $gf$ , stating its domain.  
**b** Solve the equation  $gf(x) = 10$ .

15 The functions  $f$  and  $g$  are defined by

$$f: x \rightarrow 2(x+1), x \in \mathbb{R} \qquad g: x \rightarrow x^2 - 9, x \in \mathbb{R}$$

- a** Express  $gf$  in terms of  $x$  and state its domain and range.  
**b** Sketch the graph of  $y = gf(x)$ , showing the coordinates of any points of intersection with the coordinate axes.

The equation  $gf(x) - 2f(x) = a$ , where  $a$  is a constant, has no real roots.

- c** Show that  $a < -10$ .