

- 1 Show that

$$\int_2^7 \frac{8}{4x-3} dx = \ln 25. \quad (4)$$

- 2 Given that $y = \frac{\pi}{4}$ when $x = 1$, solve the differential equation

$$\frac{dy}{dx} = x \sec y \operatorname{cosec}^3 y. \quad (7)$$

- 3 a Use the trapezium rule with three intervals of equal width to find an approximate value for the integral

$$\int_0^{1.5} e^{x^2-1} dx. \quad (4)$$

- b Use the trapezium rule with six intervals of equal width to find an improved approximation for the above integral. (2)

4
$$f(x) \equiv \frac{3(2-x)}{(1-2x)^2(1+x)}.$$

- a Express $f(x)$ in partial fractions. (4)

- b Show that

$$\int_1^2 f(x) dx = 1 - \ln 2. \quad (6)$$

- 5 The rate of growth in the number of yeast cells, N , present in a culture after t hours is proportional to N .

- a By forming and solving a differential equation, show that

$$N = Ae^{kt},$$

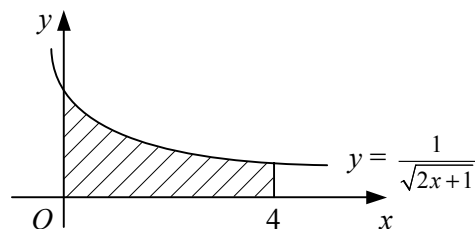
- where A and k are positive constants. (4)

Initially there are 200 yeast cells in the culture and after 2 hours there are 3000 yeast cells in the culture. Find, to the nearest minute, after how long

- b there are 10 000 yeast cells in the culture, (5)

- c the number of yeast cells is increasing at the rate of 5 per second. (4)

6



The diagram shows part of the curve with equation $y = \frac{1}{\sqrt{2x+1}}$.

The shaded region is bounded by the curve, the coordinate axes and the line $x = 4$.

- a Find the area of the shaded region. (4)

The shaded region is rotated through four right angles about the x -axis.

- b Find the volume of the solid formed, giving your answer in the form $\pi \ln k$. (5)

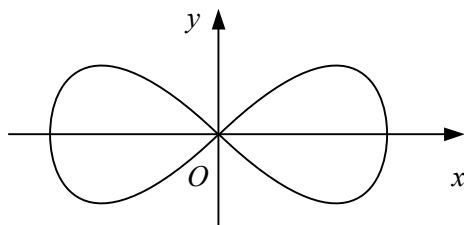
- 7 Using the substitution $u^2 = x + 3$, show that

$$\int_0^1 x\sqrt{x+3} \, dx = k(3\sqrt{3} - 4),$$

where k is a rational number to be found. (7)

- 8 a Use the identities for $\sin(A + B)$ and $\sin(A - B)$ to prove that

$$2 \sin A \cos B \equiv \sin(A + B) + \sin(A - B). \quad (2)$$



The diagram shows the curve given by the parametric equations

$$x = 2 \sin 2t, \quad y = \sin 4t, \quad 0 \leq t < \pi.$$

- b Show that the total area enclosed by the two loops of the curve is given by

$$\int_0^{\frac{\pi}{4}} 16 \sin 4t \cos 2t \, dt. \quad (4)$$

- c Evaluate this integral. (5)

9
$$f(x) \equiv \frac{x^2 - 22}{(x+2)(x-4)}.$$

- a Find the values of the constants A , B and C such that

$$f(x) \equiv A + \frac{B}{x+2} + \frac{C}{x-4}. \quad (3)$$

The finite region R is bounded by the curve $y = f(x)$, the coordinate axes and the line $x = 2$.

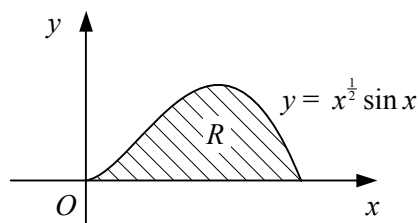
- b Find the area of R , giving your answer in the form $p + \ln q$, where p and q are integers. (5)

10 a Find $\int \sin^2 x \, dx$. (4)

- b Use integration by parts to show that

$$\int x \sin^2 x \, dx = \frac{1}{8} (2x^2 - 2x \sin 2x - \cos 2x) + c,$$

where c is an arbitrary constant. (4)



The diagram shows the curve with equation $y = x^{\frac{1}{2}} \sin x$, $0 \leq x \leq \pi$.

The finite region R , bounded by the curve and the x -axis, is rotated through 2π radians about the x -axis.

- c Find the volume of the solid formed, giving your answer in terms of π . (3)