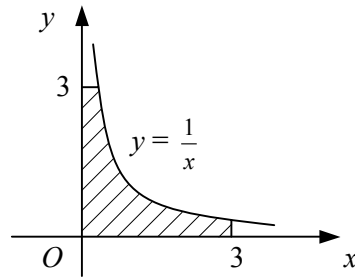


1



The diagram shows the curve with equation $y = \frac{1}{x}$, $x > 0$.

The shaded region is bounded by the curve, the lines $x = 3$ and $y = 3$ and the coordinate axes.

a Show that the area of the shaded region is $1 + \ln 9$. (5)

b Find the volume of the solid generated when the shaded region is rotated through 360° about the x -axis, giving your answer in terms of π . (5)

2 Given that

$$I = \int_0^4 x \sec\left(\frac{1}{3}x\right) dx,$$

a find estimates for the value of I to 3 significant figures using the trapezium rule with

- i** 2 strips,
- ii** 4 strips,
- iii** 8 strips. (6)

b Making your reasoning clear, suggest a value for I correct to 3 significant figures. (2)

3 The temperature in a room is 10°C . A heater is used to raise the temperature in the room to 25°C and then turned off. The amount by which the temperature in the room exceeds 10°C is $\theta^\circ\text{C}$, at time t minutes after the heater is turned off.

It is assumed that the rate at which θ decreases is proportional to θ .

a By forming and solving a suitable differential equation, show that

$$\theta = 15e^{-kt},$$

where k is a positive constant. (6)

Given that after half an hour the temperature in the room is 20°C ,

b find the value of k . (3)

The heater is set to turn on again if the temperature in the room falls to 15°C .

c Find how long it takes before the heater is turned on. (3)

4 a Find the values of the constants p , q and r such that

$$\sin^4 x \equiv p + q \cos 2x + r \cos 4x. \quad (4)$$

b Hence, evaluate

$$\int_0^{\frac{\pi}{2}} \sin^4 x \, dx,$$

giving your answer in terms of π . (4)

- 5 a Find the general solution of the differential equation

$$\frac{dy}{dx} = xy^3. \quad (4)$$

- b Given also that $y = \frac{1}{2}$ when $x = 1$, find the particular solution of the differential equation, giving your answer in the form $y^2 = f(x)$. (3)

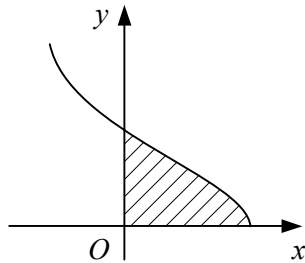
- 6 a Show that, using the substitution $x = e^u$,

$$\int \frac{2 + \ln x}{x^2} dx = \int (2 + u)e^{-u} du. \quad (3)$$

- b Hence, or otherwise, evaluate

$$\int_1^e \frac{2 + \ln x}{x^2} dx. \quad (6)$$

7



The diagram shows the curve with parametric equations

$$x = \cos 2t, \quad y = \tan t, \quad 0 \leq t < \frac{\pi}{2}.$$

The shaded region is bounded by the curve and the coordinate axes.

- a Show that the area of the shaded region is given by

$$\int_0^{\frac{\pi}{4}} 4 \sin^2 t \, dt. \quad (4)$$

- b Hence find the area of the shaded region, giving your answer in terms of π . (4)

- c Write down expressions in terms of $\cos 2A$ for

i $\sin^2 A$,

ii $\cos^2 A$,

- and hence find a cartesian equation for the curve in the form $y^2 = f(x)$. (4)

8

$$f(x) \equiv \frac{6 - 2x^2}{(x+1)^2(x+3)}.$$

- a Find the values of the constants A , B and C such that

$$f(x) \equiv \frac{A}{(x+1)^2} + \frac{B}{x+1} + \frac{C}{x+3}. \quad (4)$$

The curve $y = f(x)$ crosses the y -axis at the point P .

- b Show that the tangent to the curve at P has the equation

$$14x + 3y = 6. \quad (5)$$

- c Evaluate

$$\int_0^1 f(x) \, dx,$$

- giving your answer in the form $a + b \ln 2 + c \ln 3$ where a , b and c are integers. (5)