C3 NUMERICAL METHODS — Worksheet A

1 Show in each case that there is a root of the equation $f(x) = 0$ in the given interval.

a $f(x) = x^3 + 3x - 7$	$(1, 2)$	**b** $f(x) = 5\cos x - 3x$	$(0.5, 1)$
c $f(x) = 2e^x + x + 5$	$(-6, -5)$	**d** $f(x) = x^4 - 5x^2 + 1$	$(2.1, 2.2)$
e $f(x) = \ln(4x - 1) + x^2$	$(0.4, 0.5)$	**f** $f(x) = e^{-x} - 9\cos 4x$	$(10, 11)$

2 Given that $|N| \leq 5$, find in each case the integer N such that there is a root of the equation $f(x) = 0$ in the interval $(N, N + 1)$.

a $f(x) = x^3 - 3\sqrt{x} - 4$ **b** $f(x) = x\ln x - \frac{12}{x}$ **c** $f(x) = 2x^5 + 4x + 15$

d $f(x) = e^{x-1} + 4x - 2$ **e** $f(x) = e^x - 3\sin x$ **f** $f(x) = \tan(0.1x) + x - 6$

3 Show in each case that there is a root of the given equation in the given interval.

a $x^3 = 12 - \frac{x}{4}$	$[2, 3]$	**b** $12e^x = 9 - 4x$	$[-1, 0]$
c $10\ln 3x = 5 - 7x^2$	$[0.47, 0.48]$	**d** $\sin 4x = 7e^x$	$[-6.5, -6]$
e $4^x = 3x + 10$	$[-4, -3]$	**f** $\tan(\frac{1}{2}x) = 2x - 1$	$[2.6, 2.7]$

4 In each case there is a root of the equation $f(x) = 0$ in the given interval.
Find the integer, a, such that this root lies in the interval $(\frac{a}{10}, \frac{a+1}{10})$.

a $f(x) = x^4 + \frac{3}{x} - 5$	$(1, 2)$	**b** $f(x) = x - \ln(6 + x^2)$	$(2, 3)$
c $f(x) = 5x^3 - 3x^2 + 11$	$(-2, -1)$	**d** $f(x) = \frac{8}{x} - \cos x$	$(11, 12)$
e $f(x) = \operatorname{cosec} x + \sqrt{x}$	$(5, 6)$	**f** $f(x) = x^2 - 7e^{2x+5}$	$(-3, -2)$

5 **a** On the same set of axes, sketch the graphs of $y = x^3$ and $y = 4 - x$.

b Hence, show that the equation $x^3 + x - 4 = 0$ has exactly one real root.

c Show that this root lies in the interval $(1, 1.5)$.

6 $$f : x \to x\ln x - 1,\ x \in \mathbb{R},\ x > 0.$$

a On the same set of axes, sketch the curves $y = \ln x$ and $y = \frac{1}{x}$.

b Hence show that the equation $f(x) = 0$ has exactly one real root.

The real root of $f(x) = 0$ is α.

c Find the integer n such that $n < \alpha < n + 1$.

7 **a** On the same set of axes, sketch the curves $y = e^x$ and $y = 5 - x^2$.

b Hence show that the equation $e^x + x^2 - 5 = 0$ has exactly one negative and one positive real root.

c Show that the negative root lies in the interval $(-3, -2)$.

The positive root, α, is such that $\frac{n}{10} < \alpha < \frac{n+1}{10}$, where n is an integer.

d Find the value of n.