

- 1 a e.g. $x = \frac{1}{8} \Rightarrow \sqrt[3]{x} = \frac{1}{2}, \frac{1}{2} > \frac{1}{8}$
[any value of x in the interval $0 < x < 1$]
- b e.g. $n = 7 \Rightarrow n^3 - n + 7 = 7(49 - 1 + 1)$ which is divisible by 7
[many other examples]
- 2 assume $\sqrt{\pi}$ is rational $\Rightarrow \sqrt{\pi} = \frac{p}{q}, p, q \in \mathbb{Z}$
 $\Rightarrow \pi = \frac{p^2}{q^2}, p^2, q^2 \in \mathbb{Z} \therefore \pi$ rational
 \Rightarrow contradiction $\therefore \sqrt{\pi}$ irrational
- 3 consider $15x^2 - 11x + 2 < 0$
 $\Rightarrow (5x - 2)(3x - 1) < 0$
 $\Rightarrow \frac{1}{3} < x < \frac{2}{5}$
e.g. $x = 0.35 \Rightarrow 15x^2 - 11x + 2 = -0.0125, -0.0125 < 0$
[any value of x in the interval $\frac{1}{3} < x < \frac{2}{5}$]
- 4 a $n^2 + 2n = (2m + 1)^2 + 2(2m + 1)$
 $= 4m^2 + 4m + 1 + 4m + 2$
 $= 4m^2 + 8m + 3$
- b assume $n^2 + 2n$ even and n odd, where $n \in \mathbb{Z}$
 n odd $\Rightarrow n = 2m + 1, m \in \mathbb{Z}$
 $\Rightarrow n^2 + 2n = 4m^2 + 8m + 3 = 2(2m^2 + 4m + 1) + 1$
 $2m^2 + 4m + 1 \in \mathbb{Z} \therefore n^2 + 2n$ odd
 \Rightarrow contradiction $\therefore n$ even
- 5 a $k \cos x - \operatorname{cosec} x = 0 \Rightarrow k \cos x = \frac{1}{\sin x}$
 $\Rightarrow k \sin x \cos x = 1$
 $\Rightarrow \frac{1}{2} k \sin 2x = 1$
 $\Rightarrow \sin 2x = \frac{2}{k}$
 $|\sin 2x| \leq 1 \Rightarrow \left| \frac{2}{k} \right| \leq 1$
 $\Rightarrow |k| \geq 2$
- b $3 \cos x - \operatorname{cosec} x = 0 \Rightarrow \sin 2x = \frac{2}{3}$
 $2x = 41.810, 180 - 41.810, 360 + 41.810, 540 - 41.810$
 $2x = 41.810, 138.190, 401.810, 498.190$
 $x = 20.9, 69.1, 200.9, 249.1$ (1dp)

- 6 assume $x^2 - y^2 = 1$, where $x, y \in \mathbb{Z}^+$
 $x^2 - y^2 = 1 \Rightarrow (x+y)(x-y) = 1$
 $x, y \in \mathbb{Z}^+ \Rightarrow (x+y), (x-y) \in \mathbb{Z}$ and $(x+y) > 0$
 $\therefore x+y = 1$ and $x-y = 1$
 adding $\Rightarrow 2x = 2$
 $\Rightarrow x = 1$
 $\Rightarrow y = 0$
 \Rightarrow contradiction \therefore no positive integer solutions
- 7 a false e.g. $a = \sqrt{2}, b = 1 - \sqrt{2} \Rightarrow a$ and b irrational
 and $a + b = 1$ which is rational
 [many other examples]
- b true m, n consecutive odd integers $\Rightarrow m = 2a + 1, n = 2a + 3, a \in \mathbb{Z}$
 $\Rightarrow m + n = 2a + 1 + 2a + 3 = 4a + 4 = 4(a + 1)$
 $a + 1 \in \mathbb{Z} \therefore m + n$ divisible by 4
- c false e.g. $x = \frac{5\pi}{3} \Rightarrow \cos x = \frac{1}{2}$ and $1 + \sin x = 1 - \frac{\sqrt{3}}{2}, \frac{1}{2} > 1 - \frac{\sqrt{3}}{2}$
 [any value of x of the form $2n\pi + y, n \in \mathbb{Z}, -\frac{\pi}{2} < y < 0$]
- 8 a $\log_2 3 = \frac{p}{q} \Rightarrow 2^{\frac{p}{q}} = 3$
 $\Rightarrow (2^{\frac{p}{q}})^q = 3^q$
 $\Rightarrow 2^p = 3^q$
- b assume $\log_2 3$ is rational $\Rightarrow \log_2 3 = \frac{p}{q}, p, q \in \mathbb{Z}, q \neq 0$
 $\Rightarrow 2^p = 3^q$
 2 and 3 are co-prime $\Rightarrow p = q = 0$
 \Rightarrow contradiction $\therefore \log_2 3$ is irrational
- c e.g. $a = 2, b = \sqrt{2} \Rightarrow a$ rational and b irrational
 and $\log_a b = \frac{1}{2}$ which is rational
 [many other examples]
- 9 a $y = \frac{x-2}{4x}$ swap $x = \frac{y-2}{4y}$
 $4xy = y - 2$
 $y(4x - 1) = -2$
 $y = \frac{2}{1-4x}$
 $f^{-1}(x) = \frac{2}{1-4x}, x \in \mathbb{R}, x \neq \frac{1}{4}$
- b $f(x) = f^{-1}(x) \Rightarrow \frac{x-2}{4x} = \frac{2}{1-4x}$
 $\Rightarrow (x-2)(1-4x) = 8x$
 $\Rightarrow 4x^2 - x + 2 = 0$
 $b^2 - 4ac = 1 - 32 = -31$
 $b^2 - 4ac < 0 \Rightarrow$ no real roots
 \therefore no real values of x for which $f(x) = f^{-1}(x)$