1 Express each of the following in the form $R\cos(x-\alpha)^\circ$, where $R>0$ and $0<\alpha<90$.
Give the values of R and α correct to 1 decimal place where appropriate.

a $\cos x^\circ + \sin x^\circ$ **b** $3\cos x^\circ + 4\sin x^\circ$

c $2\sin x^\circ + \cos x^\circ$ **d** $\cos x^\circ + \sqrt{3}\sin x^\circ$

2 Express each of the following in the given form, where $R>0$ and $0<\alpha<90$.
Give the exact value of R and the value of α correct to 1 decimal place.

a $5\cos x^\circ - 12\sin x^\circ$, $R\cos(x+\alpha)^\circ$ **b** $4\sin x^\circ + 2\cos x^\circ$, $R\sin(x+\alpha)^\circ$

c $\sin x^\circ - 7\cos x^\circ$, $R\sin(x-\alpha)^\circ$ **d** $8\cos 2x^\circ - 15\sin 2x^\circ$, $R\cos(2x+\alpha)^\circ$

3 Express each of the following in the given form, where $R>0$ and $0<\alpha<\frac{\pi}{2}$.
Give the exact value of R and the value of α correct to 2 decimal places where appropriate.

a $3\sin x - 2\cos x$, $R\sin(x-\alpha)$ **b** $3\cos x + \sqrt{3}\sin x$, $R\cos(x-\alpha)$

c $8\sin 3x + 6\cos 3x$, $R\sin(3x+\alpha)$ **d** $\cos x + \frac{1}{2}\sin x$, $R\cos(x-\alpha)$

4 Find the maximum value that each expression can take and the smallest positive value of x, in degrees, for which this occurs.

a $24\sin x - 7\cos x$ **b** $4\cos 2x + 4\sin 2x$

c $3\cos x - 5\sin x$ **d** $5\sin 3x + \cos 3x$

5 **a** Express $3\sin x^\circ - 3\cos x^\circ$ in the form $R\sin(x-\alpha)^\circ$, where $R>0$ and $0<\alpha<90$.

b Hence, describe two transformations that would map the graph of $y=\sin x^\circ$ onto the graph of $y=3\sin x^\circ - 3\cos x^\circ$.

6 By first expressing each curve in an appropriate form, sketch each of the following for x in the interval $0\le x\le 360^\circ$, showing the coordinates of any turning points.

a $y=12\cos x+5\sin x$ **b** $y=\sin x-2\cos x$

c $y=2\sqrt{3}\cos x-6\sin x$ **d** $y=9\sin x+4\cos x$

7 **a** Express $\sqrt{3}\cos x-\sin x$ in the form $R\cos(x+\alpha)$, where $R>0$ and $0<\alpha<\frac{\pi}{2}$.

b Solve the equation $\sqrt{3}\cos x-\sin x=1$ for x in the interval $0\le x\le 2\pi$, giving your answers in terms of π.

8 Solve each equation for x in the interval $0\le x\le 2\pi$, giving your answers to 2 decimal places.

a $6\sin x+8\cos x=5$ **b** $2\cos x-2\sin x=1$

c $7\sin x-24\cos x-10=0$ **d** $3\cos x+\sin x+1=0$

e $\cos 2x+4\sin 2x=3$ **f** $5\sin x-8\cos x+7=0$

9 Solve each equation for x in the interval $-180^\circ\le x\le 180^\circ$, giving your answers to 1 decimal place where appropriate.

a $\sin x+\cos x=1$ **b** $4\cos x-\sin x+2=0$

c $\cos\frac{x}{2}+5\sin\frac{x}{2}-4=0$ **d** $6\sin x=5-3\cos x$

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