C3 TRIGONOMETRY — Worksheet G

1 Find all values of x in the interval $0 \leq x \leq 360°$ for which

$$\tan^2 x - \sec x = 1.$$ **(6)**

2 **a** Express $2\cos x° + 5\sin x°$ in the form $R\cos(x - \alpha)°$, where $R > 0$ and $0 < \alpha < 90$. Give the values of R and α to 3 significant figures. **(4)**

b Solve the equation

$$2\cos x° + 5\cos x° = 3,$$

for values of x in the interval $0 \leq x \leq 360$, giving your answers to 1 decimal place. **(4)**

3 **a** Solve the equation

$$\pi - 6\arctan 2x = 0,$$

giving your answer in the form $k\sqrt{3}$. **(4)**

b Find the values of x in the interval $0 \leq x \leq 360°$ for which

$$2\sin 2x = 3\cos x,$$

giving your answers to an appropriate degree of accuracy. **(6)**

4 **a** Use the identities for $\sin(A + B)$ and $\sin(A - B)$ to prove that

$$\sin P - \sin Q \equiv 2\cos\frac{P+Q}{2}\sin\frac{P-Q}{2}.$$ **(4)**

b Hence, or otherwise, find the values of x in the interval $0 \leq x \leq 180°$ for which

$$\sin 4x = \sin 2x.$$ **(6)**

5 **a** Prove the identity

$$(2\sin\theta - \operatorname{cosec}\theta)^2 \equiv \operatorname{cosec}^2\theta - 4\cos^2\theta, \quad \theta \neq n\pi, \ n \in \mathbb{Z}.$$ **(3)**

b **i** Sketch the curve $y = 3 + 2\sec x$ for x in the interval $0 \leq x \leq 2\pi$.

ii Write down the coordinates of the point where the curve meets the y-axis.

iii Find the coordinates of the points where the curve crosses the x-axis in this interval. **(7)**

6 **a** Find the exact values of R and α, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$, for which

$$\cos x - \sin x \equiv R\cos(x + \alpha).$$ **(3)**

b Using the identity

$$\cos X + \cos Y \equiv 2\cos\frac{X+Y}{2}\cos\frac{X-Y}{2},$$

or otherwise, find in terms of π the values of x in the interval $[0, 2\pi]$ for which

$$\cos x + \sqrt{2}\cos(3x - \tfrac{\pi}{4}) = \sin x.$$ **(7)**

7 **a** Prove the identity

$$\cot 2x + \operatorname{cosec} 2x \equiv \cot x, \quad x \neq \frac{n\pi}{2}, \ n \in \mathbb{Z}.$$ **(4)**

b Hence, for x in the interval $0 \leq x \leq 2\pi$, solve the equation

$$\cot 2x + \operatorname{cosec} 2x = 6 - \cot^2 x,$$

giving your answers correct to 2 decimal places. **(6)**

8 **a** Prove that for all real values of x

$$\cos (x + 30)° + \sin x° \equiv \cos (x - 30)°.$$ **(4)**

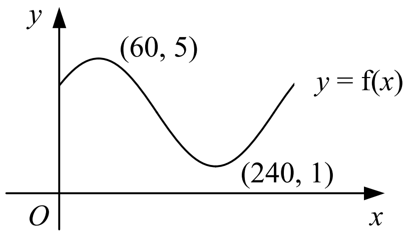
b Hence, find the exact value of $\cos 75° - \cos 15°$, giving your answer in the form $k\sqrt{2}$. **(3)**

c Solve the equation

$$3 \cos (x + 30)° + \sin x° = 3 \cos (x - 30)° + 1$$

for x in the interval $-180 \leq x \leq 180$. **(4)**

9



The diagram shows the curve $y = \text{f}(x)$ where

$$\text{f}(x) \equiv a + b \sin x° + c \cos x°, \ x \in \mathbb{R}, \ 0 \leq x \leq 360,$$

The curve has turning points with coordinates (60, 5) and (240, 1) as shown.

a State, with a reason, the value of the constant a. **(2)**

b Find the values of k and α, where $k > 0$ and $0 < \alpha < 90$, such that

$$\text{f}(x) = a + k \sin (x + \alpha)°.$$ **(3)**

c Hence, or otherwise, find the exact values of the constants b and c. **(3)**

10 **a** Prove the identity

$$\frac{1 - \cos x}{1 + \cos x} \equiv \tan^2 \frac{x}{2}, \quad x \neq (2n + 1)\pi, \ n \in \mathbb{Z}.$$ **(4)**

b Use the identity in part **a** to

i find the value of $\tan^2 \frac{\pi}{12}$ in the form $a + b\sqrt{3}$, where a and b are integers,

ii solve the equation

$$\frac{1 - \cos x}{1 + \cos x} = 1 - \sec \frac{x}{2},$$

for x in the interval $0 \leq x \leq 2\pi$, giving your answers in terms of π. **(9)**

11 **a** Prove that there are no real values of x for which

$$6 \cot^2 x - \text{cosec}\, x + 5 = 0.$$ **(4)**

b Find the values of y in the interval $0 \leq y \leq 180°$ for which

$$\cos 5y = \cos y.$$ **(6)**

12 **a** Use the identities for $\cos (A + B)$ and $\cos (A - B)$ to prove that

$$\sin A \sin B \equiv \tfrac{1}{2}[\cos (A - B) - \cos (A + B)].$$ **(2)**

b Hence, or otherwise, find the values of x in the interval $0 \leq x \leq \pi$ for which

$$4 \sin (x + \tfrac{\pi}{3}) = \text{cosec}\, (x - \tfrac{\pi}{6}),$$

giving your answers as exact multiples of π. **(7)**