

- 1 A radioactive substance is decaying such that its mass,  $m$  grams, at a time  $t$  years after initial observation is given by

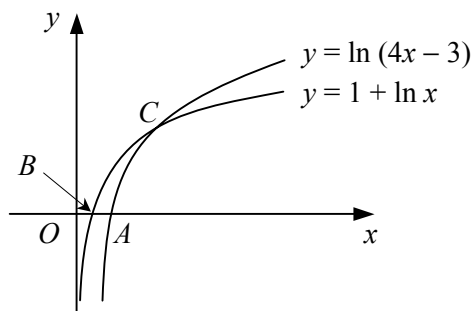
$$m = 60e^{kt},$$

where  $k$  is a constant.

Given that when  $t = 100$ ,  $m = 42$ ,

- a find the value of  $k$ , (3)  
 b find the value of  $t$  when  $m = 30$ . (2)
- 2 Solve each equation, giving your answers correct to 2 decimal places.
- a  $e^{2x} - 5.7e^{-x} = 0$  (3)  
 b  $\ln x - \ln(x - 1) = \frac{1}{2}$  (4)

3



The diagram shows the curves  $y = \ln(4x - 3)$  and  $y = 1 + \ln x$  which cross the  $x$ -axis at the points  $A$  and  $B$  respectively.

- a Find the coordinates of  $A$  and  $B$ . (4)  
 The two curves intersect at the point  $C$ .  
 b Find the exact  $x$ -coordinate of  $C$ , giving your answer in terms of  $e$ . (4)
- 4 Find, as natural logarithms, the roots of the equation
- $$2e^x + 3e^{-x} = 7. \quad (5)$$

- 5 A scientist carries out an experiment to investigate the growth of a population of flies. She introduces a colony of flies into a closed environment and uses the model that after  $t$  days the number of flies in the environment,  $N$ , is given by

$$N = 800e^{0.01t}.$$

Find, according to this model,

- a the number of flies introduced into the environment, (1)  
 b the size of the population after 20 days, (2)  
 c the least number of days after which the population will exceed 2000. (3)
- 6  $f(x) = 1 + e^{2x+1}$ .
- a Solve the equation  $f(x) = 10$ , giving your answer in the form  $a + \ln b$  where  $a$  is rational and  $b$  is an integer. (3)  
 b Find, to 3 significant figures, the  $x$ -coordinate of the point where the curve  $y = f(x)$  intersects the curve  $y = 3 - e^x$ . (5)

7 Giving your answers in exact form, solve the equations

a  $\ln(4x - 1) = 2$ , (3)

b  $7 - e^{1-3y} = 0$ . (3)

8 At time  $t = 0$ , there are 800 bacteria present in a culture. The number of bacteria present at time  $t$  hours is modelled by the continuous variable  $N$  and the relationship

$$N = ae^{bt},$$

where  $a$  and  $b$  are constants.

a Write down the value of  $a$ . (1)

Given that when  $t = 2$ ,  $N = 7200$ ,

b find the value of  $b$  in the form  $\ln k$ , (3)

c find, to the nearest minute, how long it takes for the number of bacteria present to double. (4)

9 a Simplify

$$\frac{x^2 - 4x + 3}{x^2 + x - 2}. \quad (3)$$

b Solve the equation

$$\ln(x^2 - 4x + 3) = 1 + \ln(x^2 + x - 2),$$

giving your answer in terms of  $e$ . (4)

10 Giving your answers to an appropriate degree of accuracy, solve the simultaneous equations

$$e^y + 5 - 9x = 0$$

$$y - \ln(x + 4) = 2 \quad (7)$$

11 a Describe fully the single transformation which maps the graph of  $y = e^x$  onto the graph of  $y = e^{-x}$ . (1)

b Sketch the graphs of  $y = e^{-x}$  and  $y = e^{3x+1}$  on the same diagram, showing the coordinates of any points of intersection with the coordinate axes. (4)

c Find the exact coordinates of the point of intersection of the two graphs. (3)

12 a Given that  $t = \ln x$ , find expressions in terms of  $t$  for

i  $\ln \sqrt{x}$ ,

ii  $\ln(e^2x)$ . (4)

b Hence, or otherwise, solve the equation

$$5 + \ln \sqrt{x} = \ln(e^2x). \quad (3)$$

13 A bead is projected vertically upwards in a jar of liquid with a velocity of  $13 \text{ m s}^{-1}$ . Its velocity,  $v \text{ m s}^{-1}$ , at time  $t$  seconds after projection, is given by

$$v = ce^{-kt} - 2.$$

a Find the value of  $c$ . (2)

Given that the bead has a velocity of  $7 \text{ m s}^{-1}$  after 5.1 seconds, find

b the value of  $k$  correct to 4 decimal places, (3)

c the time taken for its velocity to decrease from  $10 \text{ m s}^{-1}$  to  $4 \text{ m s}^{-1}$ . (5)