EdExcel Statistics 2

The Binomial and Poisson distributions

Section 1: The binomial distribution

Solutions to Exercise

1. $X \sim B(8, 0.6)$

(i) $P(X=0) = (0.4)^8 = 0.000655$ (3 s.f.)

(ii) $P(X=3) = \binom{8}{3}(0.6)^3(0.4)^5 = \frac{8\times 7\times 6}{1\times 2\times 3}(0.6)^3(0.4)^5 = 0.124$ (3 s.f.)

(iii) $P(X=6) = \binom{8}{6}(0.6)^6(0.4)^2 = \frac{8\times 7}{1\times 2}(0.6)^6(0.4)^2 = 0.209$ (3 s.f.)

2. $X \sim B(10, 0.7)$

(i) $P(X=0) = (0.3)^{10} = 0.00000590$ (3 s.f.)

(ii) $P(X=1) = \binom{10}{1}\times 0.7\times(0.3)^9 = 10\times 0.7\times(0.3)^9 = 0.000138$ (3 s.f.)

(iii) $P(X>1) = 1 - P(X=0) - P(X=1)$

$= 1 - 0.00000590 - 0.000138$

$= 0.9999$ (4 s.f.)

(iv) $P(X=2) = \binom{10}{2}(0.7)^2(0.3)^8 = \frac{10\times 9}{1\times 2}(0.7)^2(0.3)^8 = 0.001447$ (4 s.f.)

$P(X<3) = P(X=0) + P(X=1) + P(X=2)$

$= 0.00000590 + 0.000138 + 0.001447$

$= 0.00159$ (3 s.f.)

3. $X \sim B(10, 0.3)$

(i) $P(X \le 4) = 0.8497$

(ii) $P(X \geq 7) = 1 - P(X \leq 6)$
$= 1 - 0.9894$
$= 0.0106$

(iii) $P(5 \leq X \leq 8) = P(X \leq 8) - P(X \leq 4)$
$= 0.9999 - 0.8497$
$= 0.1502$

4. $X \sim B(15, 0.6)$
$Y \sim B(15, 0.4)$

(i) $P(X \leq 5) = P(Y \geq 10)$
$= 1 - P(Y \leq 9)$
$= 1 - 0.9662$
$= 0.0338$

(ii) $P(X \geq 8) = P(Y \leq 7)$
$= 0.7869$

(iii) $P(10 \leq X \leq 12) = P(X \leq 12) - P(X \leq 9)$
$= P(Y \geq 3) - P(Y \geq 6)$
$= 1 - P(Y \leq 2) - \left(1 - P(Y \leq 5)\right)$
$= 1 - 0.0271 - 1 + 0.4032$
$= 0.3761$

5. Let X be the number of students who pass
$X \sim B(5, 0.9)$

(i) $P(X = 5) = (0.9)^5 = 0.590$ (3 s.f.)

(ii) $P(X = 2) = \binom{5}{2} \times (0.9)^2 \times (0.1)^3 = \frac{5 \times 4}{1 \times 2} \times (0.9)^2 \times (0.1)^3 = 0.0081$

(iii) Let Y be the number of students who fail, so $Y \sim B(5, 0.1)$
$P(X \geq 3) = P(Y \leq 2)$
$= 0.9914$ (4 s.f.)

(iv) Since the probability that all five students pass is more than 0.5, this must be the greatest probability.
So the most likely number of students who pass is 5.

6. Let X be the number of orders that are dispatched on the next working day.
$X \sim B(10, 0.75)$

(i) $$\begin{aligned} P(X = 4) &= \binom{10}{4}(0.75)^4(0.25)^6 \\ &= \frac{10\times 9\times 8\times 7}{1\times 2\times 3\times 4}(0.75)^4(0.25)^6 \\ &= 0.0162 \ \text{(3 s.f.)} \end{aligned}$$

(ii) Let Y be the number of orders that are not dispatched on the next working day.
$Y \sim B(10, 0.25)$
$$\begin{aligned} P(X < 4) &= P(Y > 6) \\ &= 1 - P(Y \le 6) \\ &= 1 - 0.9965 \\ &= 0.0035 \ \text{(4 d.p.)} \end{aligned}$$

7. $X \sim B(12, 0.4)$

(i) $E(X) = np = 12 \times 0.4 = 4.8$

(ii) $$\begin{aligned} P(X = 4) &= \binom{12}{4}(0.4)^4(0.6)^8 \\ &= \frac{12\times 11\times 10\times 9}{1\times 2\times 3\times 4}(0.4)^4(0.6)^8 = 0.213 \end{aligned}$$
$$\begin{aligned} P(X = 5) &= \binom{12}{5}(0.4)^5(0.6)^7 \\ &= \frac{12\times 11\times 10\times 9\times 8}{1\times 2\times 3\times 4\times 5}(0.4)^5(0.6)^7 = 0.227 \end{aligned}$$
$$\begin{aligned} P(X = 6) &= \binom{12}{6}(0.4)^6(0.6)^6 \\ &= \frac{12\times 11\times 10\times 9\times 8\times 7}{1\times 2\times 3\times 4\times 5\times 6}(0.4)^6(0.6)^6 = 0.177 \end{aligned}$$
The most likely outcome for X is 5.

8. $X \sim B(8, 0.4)$

(i) $E(X) = np = 8 \times 0.4 = 3.2$

(ii) $P(X=3) = \binom{8}{3}(0.4)^3(0.6)^5$

$= \frac{8\times7\times6}{1\times2\times3}(0.4)^3(0.6)^5 = 0.279$

$P(X=4) = \binom{8}{4}(0.4)^4(0.6)^4$

$= \frac{8\times7\times6\times5}{1\times2\times3\times4}(0.4)^4(0.6)^4 = 0.232$

The most likely outcome for X is 3.

9. Let X be the number of white bulbs.

$X \sim B(n, 0.2)$

$P(X \geq 1) > 0.95$

$1 - P(X = 0) > 0.95$

$P(X = 0) < 0.05$

$0.8^n < 0.05$

$0.8^{13} = 0.055$ and $0.8^{14} = 0.044$

The least number of bulbs that must be selected is 14.

10. (i) $X \sim B(6, 0.15)$

(a) $P(X=0) = 0.85^6 = 0.377$ (3 s.f.)

(b) $P(X=1) = 6\times0.15\times(0.85)^5 = 0.399$ (3 s.f.)

(c) $P(X>1) = 1 - P(X=0) - P(X=1)$

$= 1-(0.85)^6 - 6\times0.15\times(0.85)^5 = 0.224$ (3 s.f.)

(d) $P(X=3) = \binom{6}{3}(0.15)^3(0.85)^3$

$= \frac{6\times5\times4}{1\times2\times3}(0.15)^3(0.85)^3 = 0.0415$ (3 s.f.)

(ii) $E(X) = np = 6\times0.15 = 0.9$

(iii) Let Y be the number of weeks in which I arrive with my suitcase on all flights.

$Y \sim B(4, 0.85^6)$

$P(Y=3) = 4\times(0.85^6)^3\times(1-0.85^6) = 0.134$

11. (i) $\text{Mean} = \frac{(0 \times 70) + (1 \times 28) + (2 \times 2)}{100} = 0.32$

(ii) Number of broken bottles = 32

Total number of bottles = 1600

$P(\text{bottle broken}) = \frac{32}{1600} = 0.02$

(iii) Let X be the number of broken bottles in a box

$X \sim B(16, 0.02)$

$$P(X = 2) = \binom{16}{2}(0.02)^2(0.98)^{14}$$

$$= \frac{16 \times 15}{1 \times 2}(0.02)^2(0.98)^{14}$$

$$= 0.0362 \text{ (3 s.f.)}$$