EdExcel Statistics 2

Hypothesis Tests

Section 1: Introducing Hypothesis Testing

Solutions to Exercise

1. Let p be the probability that a student gets an A or B grade.

   $H_0 : p = 0.4$
   $H_1 : p < 0.4$

   Significance level = 5%

   Let X be the number of students who get an A or B grade
   For $X \sim B(15, 0.4)$, $P(X \leq 2) = 0.0271 < 0.05$
   Reject H_0. There is evidence to suggest that the A and B pass-rate has decreased.

2. Let p be the probability of obtaining a head

   $H_0 : p = 0.5$
   $H_1 : p > 0.5$

   Significance level = 5%

   (i) Let X be the number of heads obtained
   For $X \sim B(10, 0.5)$, $P(X \geq 8) = 1 - P(X \leq 7)$
   $= 1 - 0.9453$
   $= 0.0547 > 0.05$
   Accept H_0. There is not sufficient evidence to suggest that the coin is biased towards heads.

   (ii) Let X be the number of heads obtained
   For $X \sim B(20, 0.5)$, $P(X \geq 16) = 1 - P(X \leq 15)$
   $= 1 - 0.9941$
   $= 0.0059 < 0.05$
   Reject H_0. There is evidence to suggest that the coin is biased towards heads.

   (iii) Let X be the number of heads obtained.
   Need the lowest possible value of a for which $P(X \geq a) < 0.05$
   $\Rightarrow 1 - P(X \leq a - 1) < 0.05$
   $\Rightarrow P(X \leq a - 1) > 0.95$

For B(50, 0.5), $P(X \leq 30) = 0.9405$
$P(X \leq 31) = 0.9675$

The lowest possible value of $a - 1$ is 31,
so the lowest possible value of a is 32.
The critical region is $X \geq 32$.

3. Let λ be the mean number of houses sold during the month of June.

$H_0 : \lambda = 6$
$H_1 : \lambda < 6$

Significance level = 5%

Let X be the number of houses sold in June.
For $X \sim Po(6)$, $P(X \leq 2) = 0.0620 > 0.05$
Accept H_0. There is not sufficient evidence to suggest that the mean number of houses sold during June has decreased.

4. Let p be the probability that a passenger loses his suitcase

$H_0 : p = 0.05$
$H_1 : p > 0.05$

Significance level = 5%

Let X be the number of times a passenger loses his suitcase
For $X \sim B(25, 0.05)$:

$$P(X \geq 3) = 1 - P(X \leq 2)$$
$$= 1 - 0.8729$$
$$= 0.1271 > 0.05$$

Accept H_0. There is not sufficient evidence to suggest that the true probability is greater than 0.05.

5. Let p be the probability that a seed germinates.

$H_0 : p = 0.65$
$H_1 : p \neq 0.65$

Significance level = 5%

Let X be the number of seeds that germinate, so $X \sim B(40, 0.65)$
Let Y be the number of seeds that do not germinate, so $Y \sim B(40, 0.35)$

For lower tail, need the highest possible value of a for which $P(X \leq a) < 0.025$.

$$P(X \leq a) < 0.025 \Rightarrow P(Y \geq 40 - a) < 0.025$$
$$\Rightarrow 1 - P(Y \leq 39 - a) < 0.025$$
$$\Rightarrow P(Y \leq 39 - a) > 0.975$$

For $B(40, 0.35)$, $P(Y \leq 19) = 0.9637$
$P(Y \leq 20) = 0.9827$

The lowest possible value of $39 - a$ for which the null hypothesis is rejected is is 20, so the highest possible value of a is 19.

For upper tail, need the lowest possible value of b for which $P(X \geq b) < 0.025$.

$$P(X \geq b) < 0.025 \Rightarrow P(Y \leq 40 - b) < 0.025$$

For $B(40, 0.35)$, $P(Y \leq 7) = 0.0124$
$P(Y \leq 8) = 0.0303$

The highest possible value of $40 - b$ is 7, so the lowest possible value of b is 33.

The critical region is $X \leq 19$ and $X \geq 33$.

The observed value of $X = 19$ lies in the critical region, so reject H_0. The evidence suggests that there has been a change in the germination rate.

6. Let p be the probability that a student does no fitness training or sporting activity out of school.

$H_0 : p = 0.7$
$H_1 : p < 0.7$

Significance level = 1%

Let X be the number of students who do no fitness training or sporting activity out of school, so $X \sim B(10, 0.7)$
Let Y be the number of students who do fitness training or sporting activity out of school, so $Y \sim B(10, 0.3)$.

Need the highest possible value of a for which $P(X \leq a) < 0.1$

$$\begin{aligned} P(X \le a) < 0.1 &\Rightarrow P(Y \ge 10 - a) < 0.1 \\ &\Rightarrow 1 - P(Y \le 9 - a) < 0.1 \\ &\Rightarrow P(Y \le 9 - a) > 0.9 \end{aligned}$$

For B(10, 0.3), $P(Y \le 4) = 0.8497$
$P(Y \le 5) = 0.9527$

The lowest possible value of $9 - a$ is 5, so the highest possible value of a is 4.
The critical region is $X \le 4$.

The observed value of $X = 5$ does not lie in the critical region, so accept H_0.
There is not sufficient evidence to support the criticism by the sporting groups.

7. Let λ be the mean number of colds per year.

$H_0 : \lambda = 4$
$H_1 : \lambda > 4$

Significance level = 5%

Let X be the colds in the first year in the new job.
For $X \sim Po(4)$, $P(X \ge 7) = 1 - P(X \le 6)$
$= 1 - 0.8893$
$= 0.1107 > 0.05$
Accept H_0. There is not sufficient evidence to suggest that the mean number of colds per year has increased.

8. (i) Let p be the probability that a casualty has to wait more than 30 minutes

$H_0 : p = 0.3$
$H_1 : p < 0.3$

Let X be the number of patients who had to wait more than 30 minutes.
Using $X \sim B(20, 0.3)$ $P(X \le 2) = 0.0355$

At the 5% significance level:
$P(X \le 2) < 0.05$, so reject H_0. There is evidence to suggest that the proportion of patients waiting more than 30 minutes has decreased.

(ii) At the 2% significance level:
$P(X \le 2) > 0.02$, so accept H_0. There is not sufficient evidence to suggest that the proportion of patients waiting more than 30 minutes has decreased.

(iii) At the 5% level, need to find highest value of r such that $P(X \le r) < 0.05$
From tables, $P(X \le 2) = 0.0355$
$P(X \le 3) = 0.1071$
The highest value of r is 2
The critical region is $X \le 2$.

9. (i) Let p be the probability that a student gets a grade A – C.

$H_0: p = 0.6$
$H_1: p \ne 0.6$

Significance level = 5%

16 out of 20 is in the upper tail.
Let X be the number of students who got grades A – C, so $X \sim B(20, 0.6)$
Let Y be the number of students who didn't get A – C, so $Y \sim B(20, 0.4)$
$P(X \ge 16) = P(Y \le 4)$
$= 0.0510$

At the 5% significance level for a two-tailed test:
$P(X \ge 16) > 0.025$, so accept H_0. There is not sufficient evidence to suggest that the proportion of students getting grades A – C is different.

(ii) Let p be the probability that a student gets a grade A – C.

$H_0: p = 0.6$
$H_1: p > 0.6$

Significance level = 5%

At the 5% significance level for a one-tailed test:
$P(X \ge 16) = 0.0510 > 0.05$, so accept H_0. There is not sufficient evidence to suggest that the proportion of students getting grades A – C has increased.