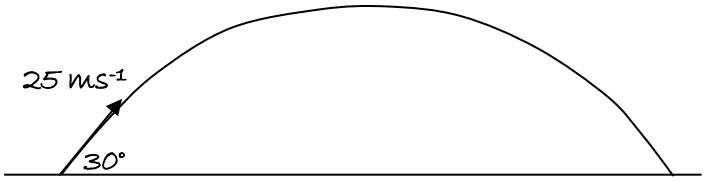
EdExcel Mechanics 2

Kinematics of a particle

Section 1: Projectiles

Solutions to Exercise

1. (i)



(ii) $u_x = 25\cos 30° = 12.5\sqrt{3}$

$u_y = 25\sin 30° = 12.5$

(iii) $v_x = 12.5\sqrt{3}$ (horizontal speed is constant)

$v_y = u_y - gt$

$= 12.5 - 9.8t$

(iv) $x = u_x t$

$= 12.5\sqrt{3}t$

$y = u_y t - \frac{1}{2}gt^2$

$= 12.5t - 4.9t^2$

2. (i) At maximum height $v_y = 0$

Vertically: $v_y^2 = u_y^2 - 2gh$

$0 = 12.5^2 - 2 \times 9.8h$

$h = \dfrac{12.5^2}{19.6} = 7.97$

Greatest height reached = 7.97 m.

(ii) The particle hits the ground when $y = 0$:

Vertically: $y = 12.5t - 4.9t^2$

$0 = t(12.5 - 4.9t)$

$t = 0$ or $t = \dfrac{12.5}{4.9}$

Time taken = 2.55 seconds.

(iii) Range = horizontal distance covered during time of flight

Horizontally: $x = 12.5\sqrt{3}t$

$$= 12.5\sqrt{3} \times \frac{12.5}{4.9}$$

$$= 55.2$$

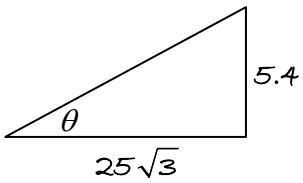
Horizontal range = 55.2 m.

3. (i) Horizontally: $v_x = 50\cos 30° = 25\sqrt{3}$

Vertically: $v_y = u + at$

$$= 50\sin 30° - 9.8 \times 2$$

$$= 5.4$$



Speed $= \sqrt{(25\sqrt{3})^2 + 5.4^2} = 43.6\ \text{ms}^{-1}$ (3 s.f.)

$\tan\theta = \dfrac{5.4}{25\sqrt{3}}$

$\theta = 7.1°$

The velocity is 43.6 ms^{-1} at an angle of 7.1° above the horizontal.

(ii) At greatest height, $v_y = 0$

Vertically: $0 = 50\sin 30° - 9.8t$

$$t = \frac{25}{9.8}$$

The time taken to reach the greatest height is 2.55 seconds (3 s.f.)

(iii) Vertically: $y = 50t\sin 30° - \frac{1}{2}gt^2$

$$= 25 \times \frac{25}{9.8} - \frac{9.8}{2}\left(\frac{25}{9.8}\right)^2$$

$$= 31.9$$

The greatest height reached = 31.9 m (3 s.f.)

4. (i) Vertically: $s = -50$, $g = -9.8$, $u = 0$, $t = ?$

$s = ut + \frac{1}{2}at^2$

$-50 = 0 + \frac{1}{2} \times -9.8t^2$

$t^2 = \dfrac{50}{4.9}$

$t = 3.19$

The time in the air is 3.19 seconds (3 s.f.)

(ii) Horizontally: $x = ut$

$$65 = u\sqrt{\frac{50}{4.9}}$$

$$u = 20.3$$

The initial speed is 20.3 ms^{-1}.

5. Vertically: $s = -19.6$, $g = -9.8$, $u = 0$, $t = ?$

$s = ut + \frac{1}{2}at^2$

$-19.6 = 0 + \frac{1}{2} \times -9.8t^2$

$t^2 = 4$

$t = 2$

It hits the ground after 2 seconds.

Horizontally: $x = ut$

$= 24.5 \times 2$

$= 49$

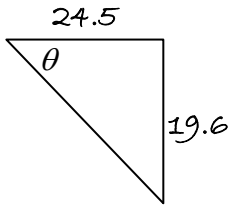
Range = 49 m.

When the ball hits ground:

Horizontally: $v_x = 24.5$

Vertically: $v_y = 0 - 9.8 \times 2$

$v_y = -19.6$



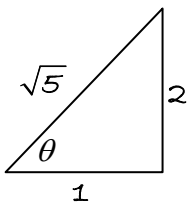
Speed $= \sqrt{24.5^2 + 19.6^2} = 31.4\ ms^{-1}$ (3 s.f.).

$\tan\theta = \frac{19.6}{24.5}$

$\theta = 38.7°$

The velocity is 31.4 ms^{-2} at an angle 38.7 below the horizontal.

6. (i)



$\tan\theta = 2$ $\sin\theta = \frac{2}{\sqrt{5}}$ $\cos\theta = \frac{1}{\sqrt{5}}$

Horizontally: $u_x = 45\cos\theta = 45 \times \frac{1}{\sqrt{5}} = 9\sqrt{5}\ ms^{-1}$

Vertically: $u_y = 45\sin\theta = 45 \times \frac{2}{\sqrt{5}} = 18\sqrt{5}$ ms^{-1}

(ii) Horizontally: $v_x = 9\sqrt{5}$ since horizontal velocity is constant

Vertically: $v_y = u_y + at$

$= 18\sqrt{5} - 9.8t$

(iii) Horizontally: $x = u_x t = 9\sqrt{5}t$

Vertically: $y = u_y t - \frac{1}{2}gt^2$

$= 18\sqrt{5}t - 4.9t^2$

(iv) Time of flight is the time at which $y = 0$

Vertically: $0 = 18\sqrt{5}t - 4.9t^2$

$0 = t(18\sqrt{5} - 4.9t)$

$t = 0$ or $t = \frac{18\sqrt{5}}{4.9}$

Time of flight = 8.21 seconds (3 s.f.)

Horizontally: $x = 9\sqrt{5}t$

$x = 9\sqrt{5} \times \frac{18\sqrt{5}}{4.9} = 165$

The range is 165 m (3 s.f.)

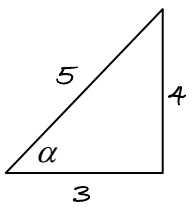
(v) At greatest height, $v_y = 0$

Vertically: $v_y^2 = u_y^2 - 2gh$

$0 = (18\sqrt{5})^2 - 2 \times 9.8h$

$h = \frac{1620}{19.6} = 82.7$

7. (i)



$\tan\alpha = \frac{4}{3}$ $\sin\alpha = \frac{4}{5}$ $\cos\alpha = \frac{3}{5}$

Horizontally: $u_x = 30\cos\alpha = 30 \times \frac{3}{5} = 18$

Vertically: $u_y = 30\sin\alpha = 30 \times \frac{4}{5} = 24$

(ii) At maximum height, $v_y = 0$

Vertically: $v_y = u_y + at$

$$0 = 24 - 9.8t$$

$$t = \frac{24}{9.8}$$

Time taken to reach highest point = 2.45 seconds (3 s.f.)

Vertically: $v_y^2 = u_y^2 - 2gh$

$$0 = 24^2 - 2 \times 9.8h$$

$$h = \frac{576}{19.6} = 29.4$$

Maximum height = 29.4 m (3 s.f.).

(iii) Time of flight is twice the time taken to reach highest point

Time of flight $= \frac{48}{9.8} = 4.90$ seconds (3 s.f.)

Horizontally: $x = 18t = 18 \times \frac{48}{9.8} = 88.2$

The range is 88.2 m (3 s.f.)

8. Horizontally: $x = 20t$

When ball reaches net, $x = 12$, so $12 = 20t \Rightarrow t = 0.6$

Vertically when ball reaches net: $y = 0 - \frac{1}{2}gt^2 = -4.9 \times 0.6^2 = -1.764$

The ball has dropped by 1.764 m, so its height is $2.8 - 1.764 = 1.036$

The net is 1 m high, so the ball clears the net by 3.6 cm.