EdExcel Mechanics 2

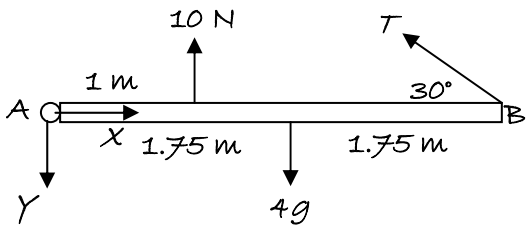
Statics of rigid bodies

Section 1: Moments of forces at an angle

Solutions to Exercise

1. (i) Moment $= 5 \times 0.25 = 1.25$ Nm
(ii) The line of action of the force must remain tangential to the steering wheel.

2.



Taking moments about A: $T \sin 30° \times 3.5 - 4g \times 1.75 + 10 \times 1 = 0$

$1.75T = 58.6$

$T = 33.5$ (3 s.f.)

Resolving vertically: $10 - Y + T \sin 30° - 4g = 0$

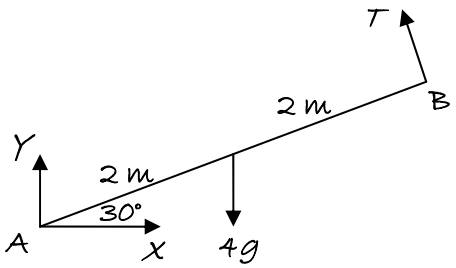
$Y = 10 + 33.486 \times \frac{1}{2} - 39.2 = -12.457$

Resolving horizontally: $X - T \cos 30° = 0$

$X = 33.486 \cos 30° = 28.999$

Magnitude of reaction force $= \sqrt{12.457^2 + 28.999^2} = 31.6$ N (3 s.f.)

3.



Taking moments about A: $4T - 4g \cos 30° \times 2 = 0$

$T = g\sqrt{3}$

$T = 16.97$ N (2 d.p.)

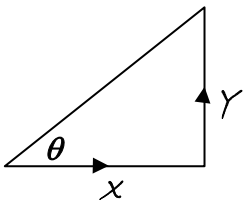
Resolving vertically: $Y - 4g + T \cos 30° = 0$

$$Y = 4g - g\sqrt{3} \times \frac{\sqrt{3}}{2} = 2.5g$$

Resolving horizontally: $X - T\sin 30° = 0$

$$X = \tfrac{1}{2}g\sqrt{3}$$

Magnitude of reaction $= g\sqrt{2.5^2 + \left(\tfrac{1}{2}\sqrt{3}\right)^2} = 25.9$ N (3 s.f.)

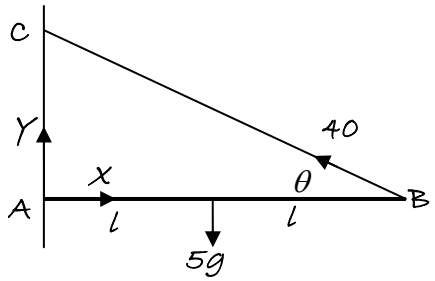


$$\tan\theta = \frac{Y}{X} = \frac{2.5}{\frac{1}{2}\sqrt{3}}$$

$\theta = 70.9°$

The direction of the reaction force is 70.9° above the horizontal.

4.

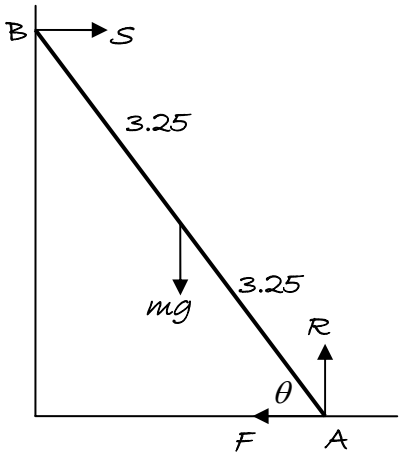


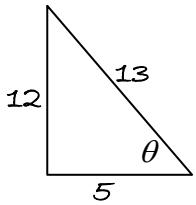
Taking moments about A: $40\sin\theta \times 2l - 5g \times l = 0$

$$\sin\theta = \frac{5g}{80}$$

$\theta = 37.8°$

5.





Resolving vertically: $R - mg = 0$

$$R = mg$$

Friction is limiting so $F = \mu R = \mu mg$

Taking moments about B:

$mg\cos\theta \times 3.25 + F\sin\theta \times 6.5 - R\cos\theta \times 6.5 = 0$

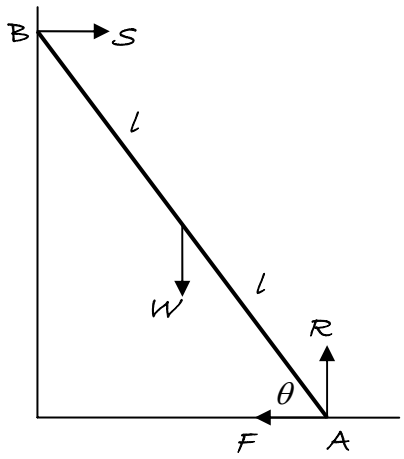
$mg\cos\theta + 2\mu mg\sin\theta - 2mg\cos\theta = 0$

$2\mu\sin\theta = \cos\theta$

$2\mu \times \frac{12}{13} = \frac{5}{13}$

$\mu = \frac{5}{24}$

6. (i)



(ii) Resolving vertically: $R - W = 0$

$R = W$

Friction is limiting: $F = \mu R = 0.5W$

Taking moments about B: $W\cos\theta \times l + F\sin\theta \times 2l - R\cos\theta \times 2l = 0$

$W\cos\theta + \frac{1}{2}W\sin\theta \times 2 - 2W\cos\theta = 0$

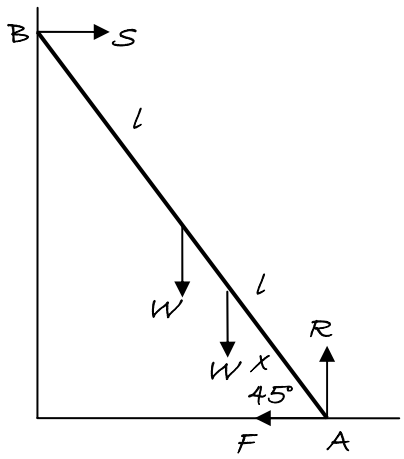
$\cos\theta = \sin\theta$

$\tan\theta = 1$

$\theta = 45°$

(iii) Resolving horizontally: $F - S = 0$

$S = 0.5W$



(iv) Resolving vertically: $R - 2W = 0$

$R = 2W$

Friction is limiting: $F = \mu R = 0.5 \times 2W = W$

Taking moments about B:

$$W\cos\theta \times l + W\cos\theta(2l - x) + F\sin\theta \times 2l - R\cos\theta \times 2l = 0$$

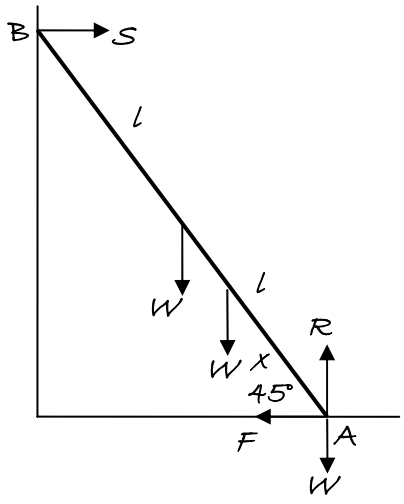
$$3\cancel{W}l\cos\theta - \cancel{W}x\cos\theta + 2\cancel{W}l\sin\theta - 4\cancel{W}l\cos\theta = 0$$

$$3l - x + 2l\tan\theta - 4l = 0$$

$$x = 3l + 2l - 4l = l$$

The ladder starts to slip when the man is halfway up the ladder.

(v)



Resolving vertically: $R - 3W = 0$

$R = 3W$

Friction is limiting: $F = \mu R = 0.5 \times 3W = \frac{3}{2}W$

Taking moments about B:

$W\cos\theta \times l + W\cos\theta(2l - x) + W\cos\theta \times 2l + F\sin\theta \times 2l - R\cos\theta \times 2l = 0$

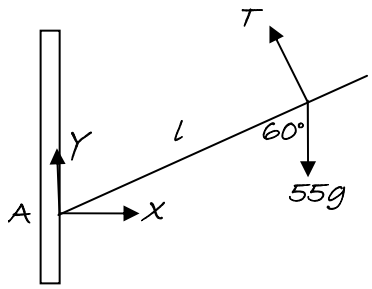
$5\cancel{W}l\cos\theta - \cancel{W}x\cos\theta + \frac{3}{2}\cancel{W} \times 2l\sin\theta - 3\cancel{W} \times 2l\cos\theta = 0$

$5l - x + 3l\tan\theta - 6l = 0$

$x = 5l + 3l - 6l = 2l$

The man can climb all the way up the ladder.

7.



Taking moments about A:

$T\cancel{l} - 55g\sin 60° \times \cancel{l} = 0$

$T = 55g\sin 60° = 27.5g\sqrt{3} = 467$ N (3 s.f.)

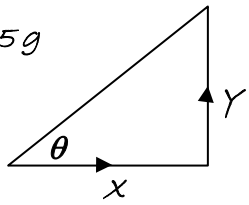
Resolving horizontally: $X - T\cos 60° = 0$

$X = 27.5g\sqrt{3} \times \frac{1}{2} = 13.75g\sqrt{3}$

Resolving vertically: $Y + T\sin 60° - 55g = 0$

$Y = 55g - 27.5g\sqrt{3} \times \frac{\sqrt{3}}{2} = 13.75g$

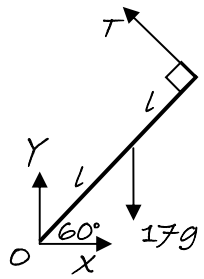
Reaction force $= g\sqrt{(13.75\sqrt{3})^2 + (13.75)^2} = 269.5$ N



$\tan\theta = \frac{Y}{X} = \frac{13.75g}{13.75g\sqrt{3}} = \frac{1}{\sqrt{3}}$

$\theta = 30°$

8. (i)



Taking moments about O: $T \times 2\cancel{l} - 17g\cos 60° \times \cancel{l} = 0$

$2T = 17g \times \frac{1}{2}$

$T = 41.65$

Tension in rope is 41.65 N

Resolving horizontally: $X - T\sin 60° = 0$

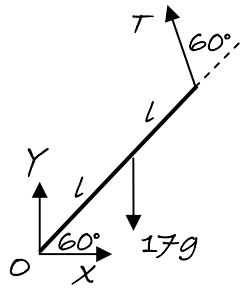
$$X = 41.65 \times \frac{\sqrt{3}}{2} = 20.825\sqrt{3}$$

Resolving vertically: $Y + T\cos 60° - 17g = 0$

$$Y = 17 \times 9.8 - 41.65 \times \tfrac{1}{2} = 145.775$$

Reaction force $= \sqrt{\left(20.825\sqrt{3}\right)^2 + 145.775^2} = 150$ N (3 s.f.)

(ii)



Taking moments about O: $T \times 2l\sin 60° - 17g\cos 60° \times l = 0$

$$2T \times \tfrac{1}{2}\sqrt{3} = 17g \times \tfrac{1}{2}$$

$$T = \frac{83.3}{\sqrt{3}} = 48.1$$

Tension in rope is 48.1 N (3 s.f.)

Resolving horizontally: $X - T\sin 30° = 0$

$$X = \frac{83.3}{\sqrt{3}} \times \frac{1}{2} = \frac{41.65}{\sqrt{3}}$$

Resolving vertically: $Y + T\cos 30° - 17g = 0$

$$Y = 17 \times 9.8 - \frac{83.3}{\sqrt{3}} \times \frac{\sqrt{3}}{2} = 124.95$$

Reaction force $= \sqrt{\left(\frac{41.65}{\sqrt{3}}\right)^2 + 124.95^2} = 127$ N (3 s.f.)