EdExcel Mechanics 2

Kinematics of a particle

Section 2: General motion

Solutions to Exercise

1. $s = t^3 + 2t^2 + 3t + 4$

$$v = \frac{ds}{dt} = 3t^2 + 4t + 3$$

$$a = \frac{dv}{dt} = 6t + 4$$

When $t = 2$, $v = 3\times 2^2 + 4\times 2 + 3 = 23$

$a = 6\times 2 + 4 = 16$

2. (i) When $t = 0$, $s = -2$, so the initial displacement = -2 m.

$s = 2t^2 + 3t - 2$

$$v = \frac{ds}{dt} = 4t + 3$$

When $t = 0$, $v = 3$ so the initial velocity is 3 ms^{-1}.

(ii) $v = 0 \quad \Rightarrow 4t + 3 = 0 \quad \Rightarrow t = -0.75$

Since this is negative, there are no times for which the velocity is zero.

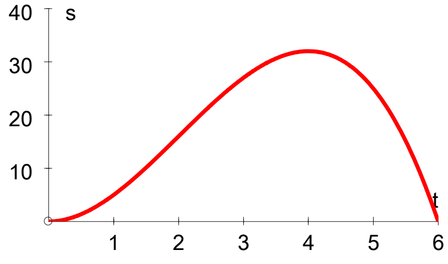
(iii) When $s = 0$, $2t^2 + 3t - 2 = 0$

$(2t - 1)(t + 2) = 0$

$t = \frac{1}{2}$ or -2

The particle is at the origin when $t = \frac{1}{2}$.

3. (i) $s = 6t^2 - t^3 = t^2(6 - t)$



$$v = \frac{ds}{dt} = 12t - 3t^2 = 3t(4 - t)$$

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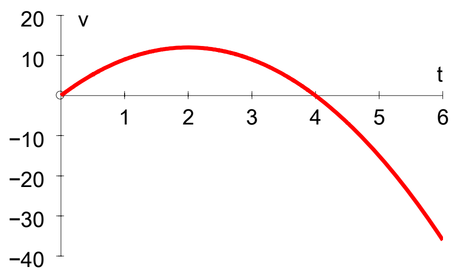

(ii) The particle is at O when $t = 0$ and when $t = 6$.

(iii) The greatest displacement is when the velocity is zero.

$3t(4-t)=0$

$t=0$ or $t=4$

From the graph, the greatest displacement is when $t = 4$

$s=4^2(6-4)=32$

The greatest displacement is 32 m.

(iv) The greatest positive speed is when the acceleration is zero

$a=\dfrac{dv}{dt}=12-6t$

When $a = 0$, $t = 2$

When $t = 2$, $v=3\times 2(4-2)=12$

The greatest negative speed in the time interval is when $t = 6$ (from the graph).

When $t = 6$, $v=3\times 6(4-6)=-36$

So the greatest speed in the time interval is 36 ms^{-1}.

4. $v=2t^3-9t^2$

$s=\int\left(2t^3-9t^2\right)dt=\frac{1}{2}t^4-3t^3+c$

When $t = 0$, $s = 20 \Rightarrow 20=c$

$s=\frac{1}{2}t^4-3t^3+20$

$a=\dfrac{dv}{dt}=6t^2-18t$

When acceleration is zero, $6t^2-18t=0$

$6t(t-3)=0$

$t=0$ or $t=3$

The acceleration is zero when $t = 0$ and when $t = 3$.

5. $a=6-2t$

$v=\int(6-2t)dt=6t-t^2+c$

When $t = 0$, $v = 0 \Rightarrow c=0$

$v=6t-t^2=t(6-t)$

The vehicle comes to rest again when $t = 6$, so it reaches point B when $t = 6$.

$s = \int (6t - t^2)\,dt = 3t^2 - \frac{1}{3}t^3 + k$
When $t = 0, s = 0 \Rightarrow k = 0$
$s = 3t^2 - \frac{1}{3}t^3$
When $t = 6,\ s = 3 \times 6^2 - \frac{1}{3} \times 6^3 = 36$
The distance AB is 36 m.

At greatest speed, acceleration is zero $\Rightarrow t = 3$
When $t = 3,\ v = 3(6-3) = 9$
The greatest speed is 9 ms^{-1}.

6. (i) $a = 6t - 4$

$v = \int (6t - 4)\,dt = 3t^2 - 4t + c$
When $t = 0,\ v = 0 \Rightarrow c = 0$
$v = 3t^2 - 4t$
$s = \int (3t^2 - 4t)\,dt = t^3 - 2t^2 + k$
When $t = 0, s = 0 \Rightarrow k = 0$
$s = t^3 - 2t^2$

(ii) When $s = 0,\ t^3 - 2t^2 = 0$
$t^2(t-2) = 0$
$t = 0$ or $t = 2$
The particle is at the origin when $t = 0$ and when $t = 2$.

(iii) The particle changes direction when $v = 0 \Rightarrow 3t^2 - 4t = 0$
$\Rightarrow t(3t - 4) = 0$
$\Rightarrow t = 0$ or $t = \frac{4}{3}$
The particle does not change direction in the first second.
When $t = 1,\ s = 1^3 - 2 \times 1^2 = -1$
so the distance travelled in the first second is 1 m.

7. $a = k(1 + 3t^2)$
When $t = 3,\ a = 14 \Rightarrow 14 = k(1 + 3 \times 3^2)$
$\Rightarrow 14 = 28k$
$\Rightarrow k = \frac{1}{2}$

$a = \frac{1}{2}(1 + 3t^2)$
$v = \int \frac{1}{2}(1 + 3t^2)\,dt = \frac{1}{2}t + \frac{1}{2}t^3 + c$

When $t = 3$, $v = 25 \Rightarrow 25 = \frac{1}{2}\times 3 + \frac{1}{2}\times 3^3 + c$

$\Rightarrow 25 = 15 + c$

$\Rightarrow c = 10$

$v = \frac{1}{2}t + \frac{1}{2}t^3 + 10$

Initial velocity = 10 ms^{-1}.

8. $\underline{r} = 2t^3\underline{i} + 3t^2\underline{j}$

$\underline{v} = \frac{d\underline{r}}{dt} = 6t^2\underline{i} + 6t\underline{j}$

$\underline{a} = \frac{d\underline{v}}{dt} = 12t\underline{i} + 6\underline{j}$

When t = 3, $\underline{v} = 6\times 3^2\underline{i} + 6\times 3\underline{j} = 54\underline{i} + 18\underline{j}$

$|\underline{v}| = \sqrt{54^2 + 18^2} = 56.9$

When t = 3, $\underline{a} = 12\times 3\underline{i} + 6\underline{j} = 36\underline{i} + 6\underline{j}$

$|\underline{a}| = \sqrt{36^2 + 6^2} = 36.5$

9. $\underline{a} = 2t\underline{i} + 3\underline{j}$

$\underline{v} = \int \underline{a}\,dt = t^2\underline{i} + 3t\underline{j} + \underline{c}$

When t = 0, $\underline{v} = 5\underline{j} \quad \Rightarrow \underline{c} = 5\underline{j}$

$\underline{v} = t^2\underline{i} + 3t\underline{j} + 5\underline{j}$

$= t^2\underline{i} + (3t+5)\underline{j}$

When t = 3, $\underline{v} = 3^2\underline{i} + (3\times 3 + 5)\underline{j}$

$= 9\underline{i} + 14\underline{j}$

$\underline{v} = t^2\underline{i} + (3t+5)\underline{j}$

$\underline{r} = \int \underline{v}\,dt = \frac{1}{3}t^3\underline{i} + (\frac{3}{2}t^2 + 5t)\underline{j} + \underline{k}$

When t = 0, $\underline{r} = 0 \quad \Rightarrow \underline{k} = 0$

$\underline{r} = \frac{1}{3}t^3\underline{i} + (\frac{3}{2}t^2 + 5t)\underline{j}$

When t = 3, $\underline{r} = \frac{1}{3}\times 3^3\underline{i} + (\frac{3}{2}\times 3^2 + 5\times 3)\underline{j}$

$= 9\underline{i} + 28.5\underline{j}$

10. $\underline{r} = 6t\underline{i} - 4t^2\underline{j}$

$\underline{v} = \frac{d\underline{r}}{dt} = 6\underline{i} - 8t\underline{j}$

$\underline{a} = \frac{d\underline{v}}{dt} = -8\underline{j}$

Magnitude of acceleration is 8.

11. (i) $\underline{r} = (2t-1)\underline{i} - t^2\underline{j}$

$\underline{v} = \frac{d\underline{r}}{dt} = 2\underline{i} - 2t\underline{j}$

(ii) When t = 0, $\underline{v} = 2\underline{i}$, so the initial direction of motion is in the $\underline{i}$ direction.

(iii) $\underline{v} = 2\underline{i} - 2t\underline{j}$

$\underline{a} = \frac{d\underline{v}}{dt} = -2\underline{j}$

The acceleration is constant since it is independent of t.

(iv) Since the acceleration is zero in the $\underline{i}$ direction, the speed in the $\underline{i}$ direction is constant. The direction of motion can never be in the $\underline{j}$ direction, since the speed in the $\underline{i}$ direction is never zero.

12. $\underline{a} = 11\underline{i} + 5\underline{j}$

$\underline{v} = \int \underline{a}\,dt = 11t\underline{i} + 5t\underline{j} + \underline{c}$

When t = 0, $\underline{v} = 0 \quad \Rightarrow \underline{c} = 0$

$\underline{v} = 11t\underline{i} + 5t\underline{j}$

$\underline{r} = \int \underline{v}\,dt = \frac{11}{2}t^2\underline{i} + \frac{5}{2}t^2\underline{j} + \underline{k}$

When t = 0, $\underline{r} = 3\underline{j} \quad \Rightarrow \underline{k} = 3\underline{j}$

$\underline{r} = \frac{11}{2}t^2\underline{i} + \frac{5}{2}t^2\underline{j} + 3\underline{j}$

$= \frac{11}{2}t^2\underline{i} + \left(\frac{5}{2}t^2 + 3\right)\underline{j}$