

Half Term 2C (ANSWERS)	Week 1
<p>1</p> $\frac{6\sqrt{3}-4}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}}$ $= \frac{10+8\sqrt{3}}{1}$ $10 + 8\sqrt{3}$	
<p>2</p> <p><i>Repeated root</i> $b^2 - 4ac = 0$</p> $p^2 - 4 \times (p - 3) \times 3 = 0$ $p^2 - 12p + 36 = 0$ $(p - 6)^2 = 0$ $p = 6$	
<p>3</p> <p><i>No real roots</i> $b^2 - 4ac < 0$</p> $(2p)^2 - 4 \times p \times 3 < 0$ $4p^2 - 12p < 0$ $4p(p - 3) < 0$ $0 < p < 3$	
<p>4</p> $1 - (1 - \cos^2\theta) - 2\cos\theta = 0$ $\cos^2\theta - 2\cos\theta = 0$ $\cos\theta(\cos\theta - 2) = 0$ $\cos\theta = 0 \quad \cos\theta = 2 \text{ (no solutions)}$ $\theta = 90^\circ, 270^\circ, 450^\circ, 630^\circ$	
<p>5</p> $2x^2 + x - 3$	

$$\begin{aligned} 1 \quad & \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}} \\ & = \frac{8+2\sqrt{15}}{2} \\ & 4 + \sqrt{15} \end{aligned}$$

$$\begin{aligned} 2 \quad & \text{Repeated root } b^2 - 4ac = 0 \\ & (k+6)^2 - 4 \times 8 \times k = 0 \\ & k^2 + 12k + 36 - 32k = 0 \\ & k^2 - 20k + 36 = 0 \\ & (k-2)(k-18) = 0 \\ & k = 2 \quad k = 18 \end{aligned}$$

$$\begin{aligned} 3 \quad & \text{No real roots } b^2 - 4ac < 0 \\ & (2p)^2 - 4 \times 1 \times 1 < 0 \\ & 4p^2 - 4 < 0 \\ & 4(p^2 - 1) < 0 \\ & -1 < p < 1 \end{aligned}$$

$$\begin{aligned} 4 \quad & \cos^2\theta - (1 - \cos^2\theta) = -0.5 \\ & 2\cos^2\theta = 0.5 \\ & \cos\theta = \pm\sqrt{0.25} \\ & \cos\theta = \pm 0.5 \\ & \theta = 60^\circ, 120^\circ, 240^\circ, 300^\circ \end{aligned}$$

$$5 \quad 3x^2 - 2x - 1$$

1

$$2^x \times 2^{-2} \times 2^3 = 2^7$$
$$x - 2 + 3 = 7$$
$$x + 1 = 7$$
$$x = 6$$

2

Repeated root $b^2 - 4ac = 0$

$$k^2 - 4 \times 9 \times (k - 5) = 0$$
$$k^2 - 36k + 180 = 0$$
$$(k - 6)(k - 30) = 0$$
$$k = 6 \quad k = 30$$

3

Real and distinct roots $b^2 - 4ac > 0$

$$(p)^2 - 4 \times 3 \times 3 > 0$$
$$p^2 - 36 > 0$$
$$(p - 6)(p + 6) < 0$$
$$p < -6 \text{ or } p > 6$$

4

$$3 \frac{\sin\theta \sin\theta}{\cos\theta} = \cos\theta$$
$$3\sin^2\theta - \cos^2\theta = 0$$
$$3\sin^2\theta - (1 - \sin^2\theta) = 0$$
$$4\sin^2\theta - 1 = 0$$
$$\sin\theta = \pm 0.5 \quad \theta = 30^\circ, 150^\circ, 210^\circ, 330^\circ$$

5

$$4x^2 - 15x + 9$$

1 $3^3 \times 3^{-2} \times 3^{-x} = 3^{-4}$

$$3 - 2 - x = -4$$

$$x = 5$$

2 *Repeated root* $b^2 - 4ac = 0$

$$(p + 5)^2 - 4 \times (p - 1) \times 8 = 0$$

$$p^2 + 10p + 25 - 32p + 32 = 0$$

$$p^2 - 22p + 57 = 0$$

$$(p - 19)(p - 3) = 0$$

$$p = 3 \quad p = 19$$

3 *Real and distinct roots* $b^2 - 4ac > 0$

$$(4)^2 - 4 \times p \times (5 - p) > 0$$

$$16 - 20p + 4p^2 > 0$$

$$4(p - 4)(p - 1) < 0$$

$$p < 1 \text{ or } p > 4$$

4 $2(1 - \sin^2\theta) - 3\sin\theta = 0$

$$2\sin^2\theta + 3\sin\theta - 2 = 0$$

$$(2\sin\theta - 1)(\sin\theta + 2) = 0$$

$$\sin\theta = 0.5 \quad \sin\theta = -2 \text{ (no solutions)}$$

$$\theta = 30^\circ, 150^\circ$$

5 $2x^2 - x - 3$

$$1 \quad 2\sqrt{2} - 4\sqrt{2} + 8\sqrt{2} = 6\sqrt{2}$$

$$2 \quad \text{Repeated root } b^2 - 4ac = 0 \\ (p + 4)^2 - 4 \times (p - 1) \times 5 = 0 \\ p^2 + 8p + 16 - 20p + 20 = 0 \\ p^2 - 12p + 36 = 0 \\ (p - 6)^2 = 0 \\ p = 6$$

$$3 \quad \text{No real roots } b^2 - 4ac < 0 \\ (3(p + 1))^2 - 4 \times 1 \times (p + 1) < 0 \\ 9p^2 + 18p + 9 - 4p - 4 < 0 \\ 9p^2 + 14p + 5 < 0 \\ (9p + 5)(p + 1) < 0 \\ -1 < p < -\frac{5}{9}$$

$$4 \quad \cos^2\theta + \cos\theta = 1 - \cos^2\theta \\ 2\cos^2\theta + \cos\theta - 1 = 0 \\ (2\cos\theta - 1)(\cos\theta + 1) = 0 \\ \cos\theta = 0.5 \quad \cos\theta = -1 \\ \theta = 60^\circ, 180^\circ, 300^\circ$$

$$5 \quad x^2 + x + 2$$

$$1 \quad \frac{3\sqrt{3}-5}{\sqrt{3}-2} \times \frac{\sqrt{3}+2}{\sqrt{3}+2}$$

$$= \frac{-1+\sqrt{3}}{-1}$$

$$= 1 - \sqrt{3}$$

$$2 \quad \text{Repeated root } b^2 - 4ac = 0$$

$$(k+3)^2 - 4 \times (k-3)(k+3) = 0$$

$$k^2 + 6k + 9 - 4k^2 + 36 = 0$$

$$-3k^2 + 6k + 45 = 0$$

$$k^2 - 2k - 15 = 0$$

$$(k-5)(k+3) = 0$$

$$k = 5 \quad k = -3$$

$$3 \quad \text{Real and distinct roots } b^2 - 4ac > 0$$

$$-(1+p)^2 - 4 \times 2 \times (5-p) > 0$$

$$1 + 2p + p^2 - 40 + 8p > 0$$

$$p^2 + 10p - 39 > 0$$

$$(p+13)(p-3) < 0$$

$$p < -13 \text{ or } p > 3$$

$$4 \quad 6(1 - \cos^2\theta) + \cos\theta = 4$$

$$6\cos^2\theta - \cos\theta - 2 = 0$$

$$(2\cos\theta + 1)(3\cos\theta - 2) = 0$$

$$\cos\theta = -\frac{1}{2} \quad \cos\theta = \frac{2}{3}$$

$$\theta = 48.1^\circ, 120^\circ, 240^\circ, 312^\circ (3 \text{ s.f.})$$

$$5 \quad x^2 - 3$$

$$1 \quad 3\sqrt[3]{3} - \sqrt[3]{3} + 9\sqrt[3]{3} = 11\sqrt[3]{3}$$

$$2 \quad \begin{aligned} & \text{Repeated root } b^2 - 4ac = 0 \\ & (k + 5)^2 - 4 \times k \times k = 0 \\ & k^2 + 10k + 25 - 4k^2 = 0 \\ & -3k^2 + 10k + 25 = 0 \\ & 3k^2 - 10k - 25 = 0 \\ & (k - 5)(3k + 5) = 0 \\ & k = 5 \quad k = -\frac{5}{3} \end{aligned}$$

$$3 \quad \begin{aligned} & \text{No real roots } b^2 - 4ac < 0 \\ & (8 - 4p)^2 - 4 \times 4 \times (8 - 7p) < 0 \\ & 64 - 64p + 16p^2 - 128 + 112p < 0 \\ & 16p^2 + 48p - 64 < 0 \\ & 4(p - 1)(p - 4) < 0 \\ & 1 < p < 4 \end{aligned}$$

$$4 \quad \begin{aligned} & 3(1 - \sin^2\theta) + 5\sin\theta = 5 \\ & 3\sin^2\theta - 5\sin\theta + 2 = 0 \\ & (\sin\theta - 1)(3\sin\theta - 2) = 0 \\ & \sin\theta = 1 \quad \sin\theta = \frac{2}{3} \\ & \theta = 41.8^\circ \quad 90^\circ \quad 138^\circ \end{aligned}$$

$$5 \quad 2x^3 + x^2 - 18x - 9$$

