

Half Term 3C (ANSWERS)	Week 1
<p>1</p> $2y + 4x = 7$ $y = 3.5 - 2x \text{ Gradient} = -2$ $(y - 5) = -2(x - 1)$ $y - 5 = -2x + 2$ $2x + y = 7$	
<p>2</p> $\cos 3\theta = \frac{1}{3}$ $3\theta = 70.5^\circ, 289.5^\circ, 430.5^\circ$ $\theta = 23.5^\circ, 96.5^\circ, 144^\circ \text{ (3 s.f.)}$	
<p>3</p> $1 + 7(2x) + \frac{7 \times 6}{1 \times 2} (2x)^2 + \frac{7 \times 6 \times 5}{1 \times 2 \times 3} (2x)^3$ $1 + 14x + 84x^2 + 280x^3$	
<p>4</p> $\frac{dy}{dx} = 3x^2 - 4x + 2$ $\text{Gradient at } (-1, -6) = 3 + 4 + 2$ $= 9$	
<p>5</p> $(y = (x + 1)^3 + 2$ $y = x^3 + 3x^2 + 3x + 3$	

$$\begin{aligned}
 1 \quad & 6y + 3x = -4 \\
 & y = -1.5 - 0.5x \quad \text{Gradient} = -\frac{1}{2} \\
 & (y - 4) = -\frac{1}{2}(x + 3) \\
 & -2y + 8 = x + 3 \\
 & x + 2y = 5
 \end{aligned}$$

$$\begin{aligned}
 2 \quad & 2(1 - \cos^2 2\theta) - \cos 2\theta = 1 \\
 & 2\cos^2 2\theta + \cos 2\theta - 1 = 0 \\
 & (2\cos 2\theta - 1)(\cos 2\theta + 1) = 0 \\
 & \cos 2\theta = \frac{1}{2} \quad \cos 2\theta = -1 \\
 & 2\theta = 60^\circ, 180^\circ, 300^\circ \\
 & \theta = 30^\circ, 90^\circ, 150^\circ
 \end{aligned}$$

$$\begin{aligned}
 3 \quad & 1 + 10(-4x) + \frac{10 \times 9}{1 \times 2}(-4x)^2 + \frac{10 \times 9 \times 8}{1 \times 2 \times 3}(-4x)^3 \\
 & 1 - 40x + 720x^2 - 7680x^3
 \end{aligned}$$

$$\begin{aligned}
 4 \quad & \frac{dy}{dx} = 6x^2 - 24 \\
 & 6x^2 - 24 = 0 \\
 & x^2 - 4 = 0 \\
 & x = \pm 2 \qquad \qquad \qquad x = 2 \quad y = -32 \qquad \qquad (2, -32) \\
 & \qquad \qquad \qquad \qquad \qquad \qquad x = -2 \quad y = -32 \qquad \qquad (-2, 32)
 \end{aligned}$$

$$\begin{aligned}
 5 \quad & y = (2x)^2 - 2(2x) \\
 & y = 4x^2 - 4x
 \end{aligned}$$

1

$$\text{Gradient} = \frac{5--3}{4-2} = 4$$

$$\text{Perpendicular gradient} = -\frac{1}{4}$$

$$(y + 3) = -\frac{1}{4}(x - 2)$$

$$-4y - 12 = x - 2$$

$$x + 4y = -10$$

2

$$2(1 - \cos^2 3\theta) = 2 - \cos 3\theta$$

$$2\cos^2 3\theta - \cos 3\theta = 0$$

$$\cos 3\theta (2\cos 3\theta - 1) = 0$$

$$\cos 3\theta = 0 \quad \cos 3\theta = \frac{1}{2}$$

$$3\theta = 60^\circ, 90^\circ, 270^\circ, 300^\circ, 420^\circ, 450^\circ$$

$$\theta = 20^\circ, 30^\circ, 90^\circ, 100^\circ, 140^\circ, 150^\circ$$

3

$$2^{10} + 10(2)^9(-3x) + \frac{10 \times 9}{1 \times 2} (2)^8(-3x)^2$$

$$1024 - 15360x + 103680x^2$$

4

$$\frac{dy}{dx} = x + \frac{1}{2}x^2 - \frac{1}{4}$$

$$\frac{1}{2} + \frac{1}{8} - \frac{1}{4} = \frac{3}{8}$$

5

$$y = (-x)^3 + 2(-x)^2 - (-x) + 3$$

$$y = -x^3 + 2x^2 + x + 3$$

$$1 \quad \text{Gradient} = \frac{9-1}{-6-4} = -5$$

$$(y - 3) = -5(x - 6)$$

$$y - 3 = -5x + 30$$

$$5x + y = 33$$

$$2 \quad 2(1 - \cos^2 2\theta) = 3\cos 2\theta$$

$$2\cos^2 2\theta + 3\cos 2\theta - 2 = 0$$

$$(2\cos 2\theta - 1)(\cos 2\theta + 2) = 0$$

$$\cos 2\theta = \frac{1}{2} \quad (\cos 2\theta = -2)$$

$$2\theta = 60^\circ, 300^\circ \quad \theta = 30^\circ, 150^\circ$$

$$3 \quad \frac{9 \times 8 \times 7}{1 \times 2 \times 3} (4^6) \left(\frac{x}{2}\right)^3$$

$$43008(x^3)$$

$$4 \quad \frac{dy}{dx} = 3x + \frac{5}{2}x^2 - \frac{5}{4}$$

$$-3 + \frac{5}{2} - \frac{5}{4} = -\frac{7}{4}$$

5 *Stretch by scale factor 2 parallel to the y axis*

(-1, 4)

$$1 \quad \begin{aligned} 2y &= x + 5 \\ y &= \frac{1}{2}x + 2.5 \\ \text{Perpendicular gradient} &= -2 \\ (y - 4) &= -2(x + 2) \\ y - 4 &= -2x - 4 \\ 2x + y &= 0 \end{aligned}$$

$$2 \quad \begin{aligned} 5\cos^2 2\theta &= 4 - 3(1 - \cos^2 2\theta) \\ 5\cos^2 2\theta &= 4 - 3 + 3\cos^2 2\theta \\ 2\cos^2 2\theta &= 1 \\ \cos 2\theta &= \pm \frac{1}{\sqrt{2}} & 2\theta &= 45^\circ, 135^\circ, 225^\circ, 315^\circ \\ & & \theta &= 22.5^\circ, 67.5^\circ, 110^\circ, 158^\circ \end{aligned}$$

3 s.f

$$3 \quad \frac{10 \times 9 \times 8 \times 7}{1 \times 2 \times 3 \times 4} (3^6) \left(\frac{x}{3}\right)^4$$

$$1890(x^4)$$

$$4 \quad \begin{aligned} \frac{dy}{dx} &= 15x^2 - 4x - 3 \\ 15x^2 - 4x - 3 &= 0 \\ (3x + 1)(5x - 3) &= 0 \\ x &= -\frac{1}{3} & x &= \frac{3}{5} \end{aligned}$$

5 Stretch by scale factor $\frac{1}{2}$ parallel to the x axis

(3,-10)

1

$$5y = 2x + 10$$

$$y = \frac{2}{5}x + 2 \quad \text{Perpendicular gradient} = -\frac{5}{2}$$

$$(y - 3) = -\frac{5}{2}(x + 4)$$

$$2y - 6 = -5x - 20$$

$$5x + 2y = -14$$

2

$$\tan^2 2\theta - 3\tan 2\theta + 2 = 0$$

$$(\tan 2\theta - 1)(\tan 2\theta - 2) = 0$$

$$\tan 2\theta = 1 \quad \tan 2\theta = 2$$

$$2\theta = 45^\circ, 63.4^\circ, 225^\circ, 243.4^\circ$$

$$\theta = 22.5^\circ, 31.7^\circ, 113^\circ, 122^\circ \quad (3 \text{ s.f.})$$

3

$$\frac{8 \times 7 \times 6 \times 5}{1 \times 2 \times 3 \times 4} (2^4) \left(\frac{3x}{2}\right)^4$$

$$5670(x^4)$$

4

$$\frac{dy}{dx} = -10 + 3x^2 \quad x = -1$$

$$\text{gradient of the tangent} = -10 + 3$$

$$= -7$$

$$x = -1 \quad y = 14$$

$$\text{Equation of the tangent } (y - 14) = -7(x + 1)$$

$$y + 7x = 7$$

5

Translation by vector $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$

$(0, 7)$

$$1 \quad 4y = -3x + 5$$

$$y = \frac{3}{4}x + \frac{5}{4} \quad \text{Gradient} = -\frac{3}{4}$$

$$(y - 4) = -\frac{3}{4}(x + 4)$$

$$4y - 16 = -3x - 12$$

$$3x + 4y = 4$$

$$2 \quad (1 - \sin^2 3\theta) - 2 = \sin^2 3\theta + 3\sin 3\theta$$

$$2\sin^2 3\theta + 3\sin 3\theta + 1 = 0$$

$$(2\sin 3\theta + 1)(\sin 3\theta + 1) = 0$$

$$\sin 3\theta = -\frac{1}{2}, -1 \quad 3\theta = 210^\circ, 270^\circ, 330^\circ$$

$$\theta = 70^\circ, 90^\circ, 110^\circ$$

$$3 \quad \frac{12 \times 11 \times 10 \times 9 \times 8}{1 \times 2 \times 3 \times 4 \times 5} \left(\frac{1}{2}\right)^7 (-2x)^5$$

$$-198(x^5)$$

$$4 \quad \frac{dy}{dx} = 9x^2 + 12x - 2 \quad \text{Gradient of } y - 3x = 2 \text{ is } 3$$

$$9x^2 + 12x - 2 = 3$$

$$9x^2 + 12x - 5 = 0$$

$$(3x - 1)(3x + 5) = 0 \quad x = \frac{1}{3} \quad x = -\frac{5}{3}$$

$$5 \quad \text{Translation by vector } \begin{bmatrix} -5 \\ 2 \end{bmatrix}$$

$$(-10, 0)$$

