Half Term 4A (ANSWERS)	Week 1

1

$\int_1^2 3x^{-4} - 6x^{-3}\,dx$

$= \frac{3}{-3x^3} + \frac{6}{2x^2}$

$= \left(\frac{-1}{8} + \frac{3}{4}\right) - (-1 + 3) = -1\frac{3}{8}$

2

$\overrightarrow{AB} = \begin{bmatrix} -4 \\ 10 \end{bmatrix}$ $\quad |\overrightarrow{BM}| = \frac{1}{2}|\overrightarrow{AB}|$

$|\overrightarrow{AB}| = \sqrt{(-4)^2 + 10^2}$

$= \sqrt{116}$ $\quad |\overrightarrow{BM}| = \frac{\sqrt{116}}{2} = \sqrt{29}$

3

$\frac{1}{5}log32 - 2\,log\,4 + log64$

$= log32^{\frac{1}{5}} - 2\,log\,2^2 + log2^6$

$= log2 - 4\,log\,2 + 6log2$

$= log\ 8$

4

$(3x + 1)\,log\,3 = log\ 18$

$3x\,log\,3 + log3 = log\ 18$

$3x\,log\,3 = log\ 6$

$x = \frac{log\,6}{3\,log\,3}$

5

$(x - 3)^2 - 9 + (y - 2)^2 - 4 - 23 = 0$
$(x - 3)^2 + (y - 2)^2 = 6^2$
Centre (3, 2) Radius 6

Half Term 4A (ANSWERS)	Week 2

1

$y = 2x^{\frac{3}{2}} - 4x^{-1}$

$\frac{dy}{dx} = 3\sqrt{x} + \frac{4}{x^2}$

$x = \frac{1}{4}$ $\quad \frac{dy}{dx} = \frac{3}{2} + 64 \quad Gradient = \frac{131}{2}$

2

$\overrightarrow{AB} = (6-3)i + (10--2)j$

$= 3i + 12j$

$|\overrightarrow{AB}| = \sqrt{3^2 + 12^2}$

$= \sqrt{153}$

$= 3\sqrt{17}$

3

$log_4 64 + log_3 27$

$log_4 4^3 + log_3 3^3$

$= 3 + 3$

$= 6$

4

$(3x-2)log2 = log6$

$3xlog2 = log24$

$x = \frac{log24}{log8}$

5

$(x + 1)^2 + (y - 3)^2 = 5^2$

Intersects the x-axis when y = 0

$(x + 1)^2 + (-3)^2 = 5^2$

$x^2 + 2x + 1 + 9 = 25$

$x^2 + 2x - 15 = 0$

$(x + 5)(x - 3) = 0$ *Intersects the x- axis at x = -5 and x = 3*

Half Term 4A (ANSWERS)	Week 3

1

$f(x) = x^{-3} - 3x^4$

$\frac{dy}{dx} = -3x^{-4} - 12x^3$

$= -\left(\frac{3}{x^4} + 12x^3\right)$ $\quad x > 0$ $\quad \frac{3}{x^4} + 12x^3 > 0$ $\quad \frac{dy}{dx} < 0$

2

$tan\,30° = \frac{a}{4\sqrt{3}}$

$a = 4$

3

$3\log x + 4\log y - 2\log(xy)$

$= \log x^3 + \log y^4 - \log x^2y^2$

$= \log \frac{x^3y^4}{x^2y^2}$

$= \log xy^2$

4

$30e^{-0.4t} = 15$
$e^{-0.4t} = 0.5$
$-0.4t = \ln 0.5$
$t = \frac{\ln 0.5}{-0.4}$
$t = 1.73$ seconds

5

$x^2 + y^2 = 5^2$
$x = 1 - y$
$(1 - y)^2 + y^2 = 25$
$1 - 2y + 2y^2 - 25 = 0$
$2y^2 - 2y - 24 = 0$
$y^2 - y - 12 = 0$
$(y - 4)(y + 3) = 0$

$y = 4 \quad x = -3 \quad (-3, 4)$
$y = -3 \quad x = 4 \quad (4, -3)$

Half Term 4A (ANSWERS)	Week 4

1

$y = 2x^4 + 64x$

$\frac{dy}{dx} = 8x^3 + 64$

$8x^3 + 64 = 0$

$x = -2 \quad y = -96 \quad (-2, -96)$

2

$Magnitude\ of \begin{bmatrix} 3 \\ -4 \end{bmatrix}\ is\ \sqrt{3^2 + (-4)^2} = 5$

$5 \times 4 = 20 \qquad 4 \times \begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 12 \\ -16 \end{bmatrix}$

3

$log_3(4x + 1) = 2$

$4x + 1 = = 3$

$4x + 1 = 9$

$x = 2$

4

$15000e^{-0.3t} = 5000$

$e^{-0.3t} = \frac{1}{3}$

$t = \frac{ln\frac{1}{3}}{-0.3}$

$t = 3.66\ years$

5

$Radius = 5 \quad Centre\ (3,5)$

$(x - 3)^2 + (y - 5)^2 = 25$

Half Term 4A (ANSWERS)	Week 5

1

$\int_1^2 3x^{\frac{1}{2}} - x^{-2}\, dx$

$= 2x^{\frac{3}{2}} + \frac{1}{x}$

$\left(2 \times 2\sqrt{2} + \frac{1}{2}\right) - (2 + 1) = 4\sqrt{2} - \frac{5}{2}$

2

$\overrightarrow{AB} = \begin{bmatrix}3\\4\end{bmatrix} - \begin{bmatrix}-1\\2\end{bmatrix}$

$= \begin{bmatrix}4\\2\end{bmatrix}$ $\quad \theta = tan^{-1}\left(\frac{2}{4}\right)$

$\theta = 26.6°$

3

$2log_a 4 - log_a 4 + \frac{1}{2} log_a 16 = \frac{1}{2} log_a x$

$log_a 4 + log_a 4 = \frac{1}{2} log_a x$

$log_a 4^2 = log_a x^{\frac{1}{2}}$

$x^{\frac{1}{2}} = 16 \quad x = 256$

4

$4000 \times 1.035^t = 10000$

$1.035^t = 2.5$

$t \ln 1.035 = \ln 2.5$

t =26.6 years

5

Centre of circle $= \left(\frac{7-1}{2}, \frac{-1+5}{2}\right) = (3, 2)$

Radius of circle $= \frac{1}{2}\sqrt{8^2 + 6^2} = 5$

$(x - 3)^2 + (y - 2)^2 = 25$

$x = 0$

$9 + y^2 - 4y + 4 - 25 = 0$

$y^2 - 4y - 12 = 0$

$(y - 6)(y + 2) = 0$ $\quad$ *(0,6) (0,-2)*

Half Term 4A (ANSWERS)	Week 6

1

$$\int 9 - \frac{6}{x^2} + \frac{1}{x^4}\, dx = 9x + \frac{6}{x} - \frac{1}{3x^3} + c$$

$$\left(18 + 3 - \frac{1}{24}\right) - \left(9 + 6 - \frac{1}{3}\right) = 6\frac{7}{24}$$

2

$M = \left(\frac{2+6}{2}, \frac{5-3}{2}\right)$

$= (4, 1)$

$$\overrightarrow{CM} = \begin{bmatrix}4\\1\end{bmatrix} - \begin{bmatrix}-1\\4\end{bmatrix}$$

$$= \begin{bmatrix}5\\-3\end{bmatrix}$$

3

$$log_2(x^2 + 4) = log_2 8$$
$$x^2 + 4 = 8$$
$$x^2 = 2$$
$$x = \pm\sqrt{2} \qquad x = \sqrt{2}$$

4

$m_0 e^{-60k} = \frac{m_0}{2}$

$e^{-60k} = 0.5$

$-60k = \ln 0.5$

$k = \frac{\ln 0.5}{-60}$ $\qquad k = 0.0116$

5

$(x - 2)^2 - 4 + (y + 1)^2 - 1 - 8 = 0$

$(x - 2)^2 + (y + 1)^2 = 13$

Centre (2, -1) Gradient of line from centre to (0,2) $= \frac{3}{2}$

Gradient of tangent $= \frac{2}{3}$ $(y - 2) = \frac{2}{3}x$

$3y - 2x = 6$

Half Term 4A (ANSWERS)	Week 7

1

$y = 8x^{\frac{3}{2}} + 64x^{-1}$ $x = 4$ $y = 80$

$\frac{dy}{dx} = 12\sqrt{x} - \frac{64}{x^2}$

$x = 4$ $\frac{dy}{dx} = 12 \times 2 - \frac{64}{16}$ $y = 20x$

2

$p + 3q = 5$ $3p + 9q = 15$

$3p + 4q = 5$

$5q = 10$

$q = 2$ $p = -1$

3

$log_a(x + 3) - log_a 2 = log_a 3x$

$log_a \frac{(x+3)}{2} = log_a 3x$

$\frac{(x+3)}{2} = 3x$ $6x = x + 3$ $5x = 3$ $x = \frac{3}{5}$

4

$120 = 200e^{-10k}$
$e^{-10k} = 0.6$
$-10\,k = ln\ 0.6$
$k = \frac{ln\ 0.6}{-10}$
$k = 0.0511$

5

$(x - 1)^2 - 1 + (y - 1)^2 - 1 - 23 = 0$
$(x - 1)^2 + (y - 1)^2 = 25$
Centre (1, 1) Gradient of line from centre to (5,4) $= \frac{3}{4}$
Gradient of tangent $= \frac{4}{3}$ $(y - 4) = \frac{4}{3}(x - 5)$

$3y + 4x = 32$