

Half Term 4C (ANSWERS)	Week 1
<p>1</p> $120 = 200e^{-10k}$ $e^{-10k} = 0.6$ $-10k = \ln 0.6$ $k = \frac{\ln 0.6}{-10}$ $k = 0.0511$	
<p>2</p> $\log_a(x+3) - \log_a 2 = \log_a 3x$ $\log_a \frac{(x+3)}{2} = \log_a 3x$ $\frac{(x+3)}{2} = 3x \quad 6x = x+3 \quad 5x = 3 \quad x = \frac{3}{5}$	
<p>3</p> $p + 3q = 5 \quad 3p + 9q = 15$ $3p + 4q = 5$ $5q = 10$ $q = 2 \quad p = -1$	
<p>4</p> $\int 3x^2 - 2x + 4 \, dx = x^3 - x^2 + 4x + c$	
<p>5</p> $(x-1)^2 - 1 + (y-1)^2 - 1 - 23 = 0$ $(x-1)^2 + (y-1)^2 = 25$ <p>Centre (1, 1) Gradient of line from centre to (5, 4) = $\frac{3}{4}$</p> <p>Gradient of tangent = $\frac{4}{3}$ $(y-4) = \frac{4}{3}(x-5)$</p> $3y + 4x = 32$	

1

$$(3x + 1) \log 3 = \log 18$$

$$3x \log 3 + \log 3 = \log 18$$

$$3x \log 3 = \log 6$$

$$x = \frac{\log 6}{3 \log 3}$$

2

$$\frac{1}{5} \log 32 - 2 \log 4 + \log 64$$

$$= \log 32^{\frac{1}{5}} - 2 \log 2^2 + \log 2^6$$

$$= \log 2 - 4 \log 2 + 6 \log 2$$

$$= \log 8$$

3

$$\overrightarrow{AB} = \begin{bmatrix} -4 \\ 10 \end{bmatrix} \quad |\overrightarrow{AB}| = \sqrt{4^2 + 10^2}$$

$$= \sqrt{116}$$

$$|\overrightarrow{BM}| = \frac{\sqrt{116}}{2}$$

$$= \sqrt{29}$$

4

$$\int 10x - 3x^2 + 8 \, dx = 5x^2 - x^3 + 8x + c$$

5

$$(x - 3)^2 - 9 + (y - 2)^2 - 4 - 23 = 0$$

$$(x - 3)^2 + (y - 2)^2 = 6^2$$

Centre (3, 2) Radius 6

$$1 \quad (3x - 2)\log 2 = \log 6$$

$$3x\log 2 = \log 24$$

$$x = \frac{\log 24}{\log 8}$$

$$2 \quad \log_4 64 + \log_3 27$$

$$\log_4 4^3 + \log_3 3^3$$

$$= 3 + 3$$

$$= 6$$

$$3 \quad \overrightarrow{AB} = (6 - 3)i + (10 - -2)j$$

$$= 3i + 12j$$

$$|\overrightarrow{AB}| = \sqrt{3^2 + 12^2}$$

$$= \sqrt{153}$$

$$= 3\sqrt{17}$$

$$4 \quad \int 2x^3 - 5x^2 - 3x \, dx = \frac{1}{2}x^4 - \frac{5}{3}x^3 - \frac{3}{2}x^2 + c$$

$$5 \quad (x + 1)^2 + (y - 3)^2 = 5^2$$

Intersects the x-axis when $y = 0$

$$(x + 1)^2 + (-3)^2 = 5^2$$

$$x^2 + 2x + 1 + 9 = 25$$

$$x^2 + 2x - 15 = 0$$

$$(x + 5)(x - 3) = 0$$

Intersects the x-axis at $x = -5$ and $x = 3$

$$1 \quad \begin{aligned} 30e^{-0.4t} &= 15 \\ e^{-0.4t} &= 0.5 \\ -0.4t &= \ln 0.5 \\ t &= \frac{\ln 0.5}{-0.4} \\ t &= 1.73 \text{ seconds} \end{aligned}$$

$$2 \quad \begin{aligned} 3\log x + 4\log y - 2\log(xy) \\ &= \log x^3 + \log y^4 - \log x^2y^2 \\ &= \log \frac{x^3y^4}{x^2y^2} \\ &= \log(xy^2) \end{aligned}$$

$$3 \quad \begin{aligned} \tan 30^\circ &= \frac{a}{4\sqrt{3}} \\ a &= 4 \end{aligned}$$

$$4 \quad \begin{aligned} \int 6x^2 + 4x - 3 \, dx &= 2x^3 + 2x^2 - 3x + c \\ (16 + 8 - 6) - (2 + 2 - 3) &= 17 \end{aligned}$$

$$5 \quad \begin{aligned} x^2 + y^2 &= 5^2 \\ x &= 1 - y \\ (1 - y)^2 + y^2 &= 25 \\ 1 - 2y + 2y^2 - 25 &= 0 & y = 4 \quad x = -3 \quad (-3, 4) \\ 2y^2 - 2y - 24 &= 0 & y = -3 \quad x = 4 \quad (4, -3) \\ y^2 - y - 12 &= 0 \\ (y - 4)(y + 3) &= 0 \end{aligned}$$

$$1 \quad 15000e^{-0.3t} = 5000$$

$$e^{-0.3t} = \frac{1}{3}$$

$$t = \frac{\ln \frac{1}{3}}{-0.3}$$

$$t = 3.66 \text{ years}$$

$$2 \quad \log_3(4x + 1) = 2$$

$$4x + 1 = 3$$

$$4x + 1 = 9$$

$$x = 2$$

$$3 \quad \text{Magnitude of } \begin{bmatrix} 3 \\ -4 \end{bmatrix} \text{ is } \sqrt{3^2 + (-4)^2} = 5$$

$$5 \times 4 = 20 \quad 4 \times \begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 12 \\ 16 \end{bmatrix}$$

$$4 \quad \int 4x - 3x^2 + 1 \, dx = 2x^2 - x^3 + x + c$$

$$(8 - 8 + 2) - (0) = 2$$

$$5 \quad \text{Radius} = 5 \quad \text{Centre } (3,5)$$

$$(x - 3)^2 + (y - 5)^2 = 25$$

$$1 \quad 4000 \times 1.035^t = 10000$$

$$1.035^t = 2.5$$

$$t \ln 1.035 = \ln 2.5$$

$$t = 26.6 \text{ years}$$

$$2 \quad 2 \log_a 4 - \log_a 4 + \frac{1}{2} \log_a 16 = \frac{1}{2} \log_a x$$

$$\log_a 4 + \log_a 4 = \frac{1}{2} \log_a x$$

$$\log_a 4^2 = \log_a x^{\frac{1}{2}}$$

$$x^{\frac{1}{2}} = 16 \quad x = 256$$

$$3 \quad \overrightarrow{AB} = \begin{bmatrix} 3 \\ 4 \end{bmatrix} - \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4 \\ 2 \end{bmatrix} \quad \theta = \tan^{-1} \left(\frac{2}{4} \right)$$

$$\theta = 26.6^\circ$$

$$4 \quad \int 2x^2 + 12x + 18 \, dx = \frac{2}{3}x^3 + 6x^2 + 18x + c$$

$$\left(\frac{2}{3} + 6 + 18 \right) - \left(-\frac{2}{3} + 6 - 18 \right) = 37\frac{1}{3}$$

$$5 \quad \text{Centre of circle} = \left(\frac{7-1}{2}, \frac{-1+5}{2} \right) = (3, 2)$$

$$\text{Radius of circle} = \frac{1}{2} \sqrt{8^2 + 6^2} = 5$$

$$(x-3)^2 + (y-2)^2 = 25$$

$$x = 0$$

$$9 + y^2 - 4y + 4 - 25 = 0$$

$$y^2 - 4y - 12 = 0$$

$$(y-6)(y+2) = 0 \quad (0,6) \quad (0,-2)$$

$$1 \quad m_0 e^{-60k} = \frac{m_0}{2}$$

$$e^{-60k} = 0.5$$

$$-60k = \ln 0.5$$

$$k = \frac{\ln 0.5}{-60} \quad k = 0.0116$$

$$2 \quad \log_2(x^2 + 4) = \log_2 8$$

$$x^2 + 4 = 8$$

$$x^2 = 4$$

$$x = \pm \sqrt{4} \quad x = \pm 2$$

$$3 \quad M = \left(\frac{2+6}{2}, \frac{5-3}{2} \right)$$

$$= (4, 1)$$

$$\overrightarrow{CM} = \begin{bmatrix} 4 \\ 1 \end{bmatrix} - \begin{bmatrix} -1 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} 5 \\ -3 \end{bmatrix}$$

$$4 \quad \int 3x^2 + 10x - 2 \, dx = x^3 + 5x^2 - 2x + c$$

$$(8 + 20 - 4) - (1 + 5 - 2) = 20$$

$$5 \quad (x-2)^2 - 4 + (y+1)^2 - 1 - 8 = 0$$

$$(x-2)^2 + (y+1)^2 = 13$$

Centre (2, -1) Gradient of line from centre to (0,2) = $\frac{3}{2}$

$$\text{Gradient of tangent} = \frac{2}{3} \quad (y-2) = \frac{2}{3}x$$

$$3y - 2x = 6$$

