Half Term 5C (ANSWERS)	Week 1

1

$x(4-x)=0 \quad x=0 \quad x=4$

$\int_0^4 4x - x^2 dx$

$= \left[2x^2 - \frac{1}{3}x^3\right]$

$= \left[2\times 4^2 - \frac{1}{3}\times 4^3\right] - 0$

$= 10\frac{2}{3}$

2

$x=3 \quad y=24 \qquad 9 - \frac{1}{3}\times 27 + 3a = 24 \qquad a=8$

$\frac{dy}{dx} = 2x - x^2 + 8$

$\frac{dy}{dx} = 0 \quad 2x - x^2 + 8 = 0$

$x = -2 \quad x = 4$

3

$2\mathbf{b} - \mathbf{a} = 2i + 5j$

$|2\mathbf{b} - \mathbf{a}| = \sqrt{2^2 + 5^2}$

$= \sqrt{29}$

4

$\cos^2 x - 2\sin x \cos x + \sin^2 x + \cos^2 x + 2\sin x \cos x + \sin^2 x$

$= 2\cos^2 x + 2\sin^2 x$

$= 2(\cos^2 x + \sin^2 x)$

$= 2$

5

$9C5 \times 3^4 \times \left(\frac{1}{3}x\right)^5 = 42x^5$

The coefficient = 42

Half Term 5C (ANSWERS)	Week 2

1

$\int_{-2}^{8} 16 + 6x - x^2\, dx = \left[16x + 3x^2 - \frac{1}{3}x^3 + c\right]$

$\left[16 \times 8 + 3 \times 8^2 - \frac{1}{3} \times 8^3\right] - \left[16 \times (-2) + 3 \times (-2)^2 - \frac{1}{3} \times (-2)^3\right]$

$= 166\frac{2}{3}$

2

$\frac{dy}{dx} = 3x^2 - 10x + k$

$3 \times 2^2 - 10 \times 2 + k = 0$

$k = 8$

$y = x^3 - 5x^2 + 8x$

$x = 2 \quad y = 4$

3

$|q| = \sqrt{3^2 + 4^2}$

$|q| = 5$

$\begin{pmatrix} -15 \\ 20 \end{pmatrix}$

4

$cos^2\theta - (1 - cos^2\theta) + 0.5 = 0$

$2cos^2\theta - 0.5 = 0$

$cos\theta = \pm\frac{1}{2}$

$\theta = 60°, 120°, 240°, 300°$

5

$(1 + 2x)^7 = \ldots\ldots + 7C3 \times 1^4 \times (2x)^3 + 7C4 \times 1^3 \times (2x)^4$

$= \ldots\ldots 280x^3 + 560x^4 \ldots$

$(x - 1)(1 + 2x)^7 = \ldots\ldots 280x^4 - 560x^4$

$= -280x^4$

Half Term 5C (ANSWERS)	Week 3

1

$y = x^3 - 4x^2 + 3x$

$\int_0^1 x^3 - 4x^2 + 3x\,dx$ $\int_1^3 x^3 - 4x^2 + 3x\,dx$

$\left[\frac{1}{4}x^4 - \frac{4}{3}x^3 + \frac{3}{2}x^2 + c\right]$

$= \frac{5}{12}$ $-2\ \frac{2}{3}$ $\quad Total\ Area = 3\frac{1}{12}$

2

$6 = 2 - a + 10 \qquad a = 6$

$\frac{dy}{dx} = \frac{1}{\sqrt{x}} - 6$

$\frac{1}{\sqrt{x}} = 6 \quad x = \frac{1}{36}$

3

$a = 5\,cos30° \quad b = 5\,sin30°$

$\boldsymbol{p} = \frac{5\sqrt{3}}{2}i + \frac{5}{2}j$

4

$3sin^2\theta = cos^2\theta$
$3sin^2\theta - (1 - sin^2\theta) = 0$
$4sin^2\theta = 1$
$sin\theta = \pm\frac{1}{2}$
$\theta = 30^o, 150^o, 210^o, 330^o$

5

$10C5 \times \left(\frac{1}{3}\right)^5 \times (-3x)^5$

$= -252x^5$

Half Term 5C (ANSWERS)	Week 4

1

$\int_4^9 3x + 4\sqrt{x} + 2\,dx$

$\left[\frac{3}{2}x^2 + \frac{8}{3}x^{\frac{3}{2}} + 2x\right]$

$= \left[\frac{3}{2}9^2 + \frac{8}{3}9^{\frac{3}{2}} + 2 \times 9\right] - \left[\frac{3}{2}4^2 + \frac{8}{3}4^{\frac{3}{2}} + 2 \times 4\right] = 158\frac{1}{6}$

2

$\frac{dy}{dx} = 32x^3$

x = -0.5 *Gradient of tangent = -4*

y = -2.5 *Gradient of normal = ¼*

$y + \frac{5}{2} = \frac{1}{4}(x + \frac{1}{2})$

8y + 20 = 2x + 1

8y - 2x = -19

3

$\overrightarrow{XY} = -8i + 13j$

$\left|\overrightarrow{XY}\right| = \sqrt{8^2 + 13^2}$

$= \sqrt{233}$

4

$\frac{sin\theta}{cos\theta} + \frac{cos\theta}{sin\theta}$

$= \frac{sin^2\theta + cos^2\theta}{cos\theta sin\theta}$

$= \frac{1}{sin\theta cos\theta}$

5

$5C4 \times (2)^1 \times (-3x)^4 = 810x^4$

$5C2 \times (2)^3 \times (-3x)^2 = 720x^2$

Coefficient of the x^4 term = 720 - 810

= -90

Half Term 5C (ANSWERS)	Week 5

1

$\int_1^9 1 + 2x + \sqrt{x}\, dx$

$\left[x + x^2 + \frac{2}{3}x^{\frac{3}{2}}\right]$

$= \left[9 + 9^2 + \frac{2}{3}9^{\frac{3}{2}}\right] - \left[1 + 1^2 + \frac{2}{3}1^{\frac{3}{2}}\right] = 105\frac{1}{3}$

2

$\frac{dy}{dx} = \frac{5}{\sqrt{x}}$

$x = 4$ *Gradient of tangent* $= \frac{5}{2}$

Gradient of normal $= -\frac{2}{5}$

$y = 10$ $y - 10 = -\frac{2}{5}(x - 4)$ $5y + 2x = 58$

3

$|3i - 3j| = \sqrt{3^2 + 3^2}$

$3\sqrt{2}$

$a = \sqrt{\frac{3}{2}}$ $b = -\sqrt{\frac{3}{2}}$

4

$4cos\theta - 1 = 2sin\theta tan\theta$

$4cos\theta - 1 = 2\frac{sin^2\theta}{cos\theta}$

$4cos^2\theta - cos\theta = 2(1 - cos^2\theta)$

$6cos^2\theta - cos\theta - 2 = 0$

$cos\theta = \frac{2}{3}$ $cos\theta = -\frac{1}{2}$ $\theta = 48.2°,\ 120°,\ 240°\ 312°$

5

$12C6 \times \left(\frac{1}{2}\right)^6 \times (2x)^6 = 924x^6$

Half Term 5C (ANSWERS)	Week 6

1

$\int 4x^2 + 12x + 9\, dx = \frac{4}{3}x^3 + 6x^2 + 9x + c$

0 - - 9

= 9

2

$y = x^{\frac{5}{2}}$ $\frac{dy}{dx} = \frac{5}{2}x^{\frac{3}{2}}$

$x = 4$ gradient of tangent = 20

$y = 32$

$y - 32 = 20\,(x - 4)$

$y - 32 = 20x - 80$ $20x - y = 48$

3

Distance between A and the midpoint $= \sqrt{(3-6)^2 + (2-8)^2}$

$= \sqrt{45}$

$= 3\sqrt{5}$

Distance between A and B $= 6\sqrt{5}$

4

$y5sin\theta = 1 + 2cos^2\theta$

$5sin\theta = 1 + 2(1 - sin^2\theta)$

$2sin^2\theta + 5sin\theta - 3 = 0$

$sin\theta = \frac{1}{2}$ $(sin\theta = -3)$

$\theta = 30°, 150°$

5

$6C5 \times (2)^1 \times (-2x)^5 = -382x^5$

$6C3 \times (2)^3 \times (-2x)^3 = -1280x^3$

Coefficient of the x^5 term = 768 - 1280

= -512

Half Term 5C (ANSWERS)	Week 7

1

$5x - x^2 = 10x - 2x^2$

$x^2 - 5x = 0 \quad x = 0$ and $x = 5$

$\int_0^5 10x - 2x^2 - 5x + x^2 dx$

$\int_0^5 5x - x^2 dx = \left[\frac{5}{2}x^2 - \frac{1}{3}x^3 + c\right]$

$\frac{5}{2} \times 5^2 - \frac{1}{3} \times 5^3 = 20\frac{5}{6}$

2

$\frac{dy}{dx} = 5x^4 - 180x^2$

$5x^4 - 180x^2 = 0$

$5x^2(x^2 - 36) = 0$

$x = 0 \quad x = 6 \quad x = -6$

3

$|\overrightarrow{OA}| = \sqrt{4^2 + 2^2} \qquad |\overrightarrow{OB}| = \sqrt{8^2 + 0^2}$

$= 2\sqrt{5} \qquad = 8$

$Angle\ AOB = tan^{-1}\left(\frac{4}{2}\right)$

$= 63.4^o$

$Area = \frac{1}{2} \times 2\sqrt{5} \times 8 \times sin(63.4) \qquad Area = 16$

4

$3x - 60 = 30^o, 150^o, 390^o, 510^o, 750^o, 870^o$

$3x = 90^o, 210^o, 450^o, 570^o, 810^o, 930^o$

$x = 30^o, 70^o, 150^o, 190^o, 270^o, 310^o$

5

$11C5 \times (3)^6 \times (-\frac{1}{3}x)^5 = -1386x^5$